

Synthesizing safe coalition strategies

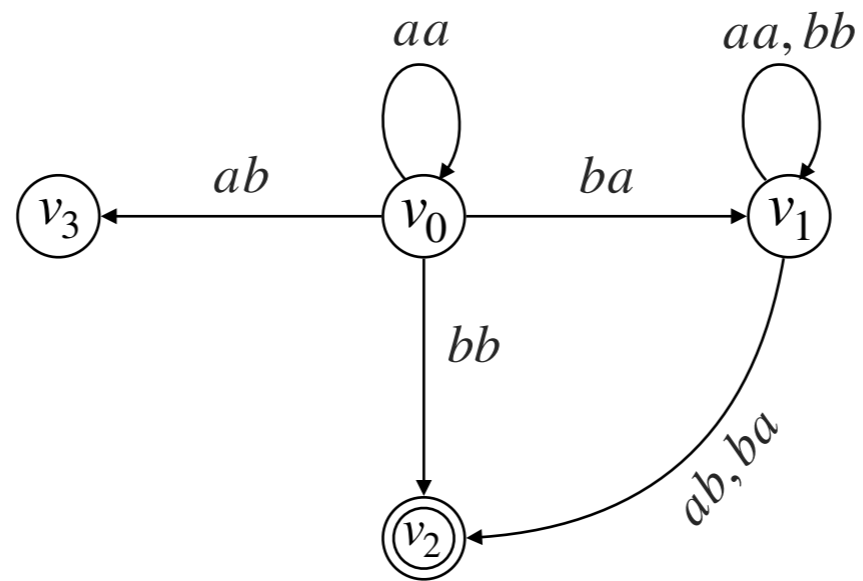
Nathalie Bertrand, Patricia Bouyer and [Anirban Majumdar](#)

Inria Rennes, France

LSV, ENS Paris-Saclay, France

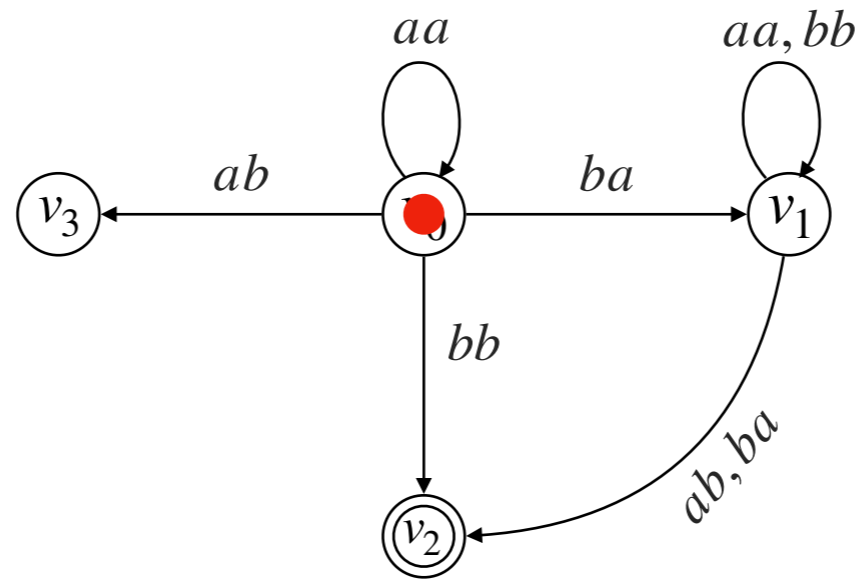
FSTTCS 2020

2-player Concurrent games on graphs



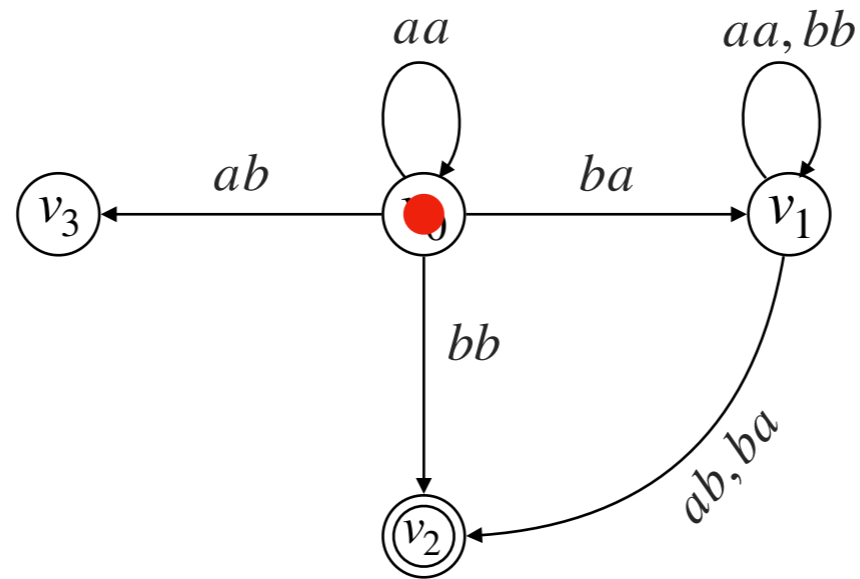
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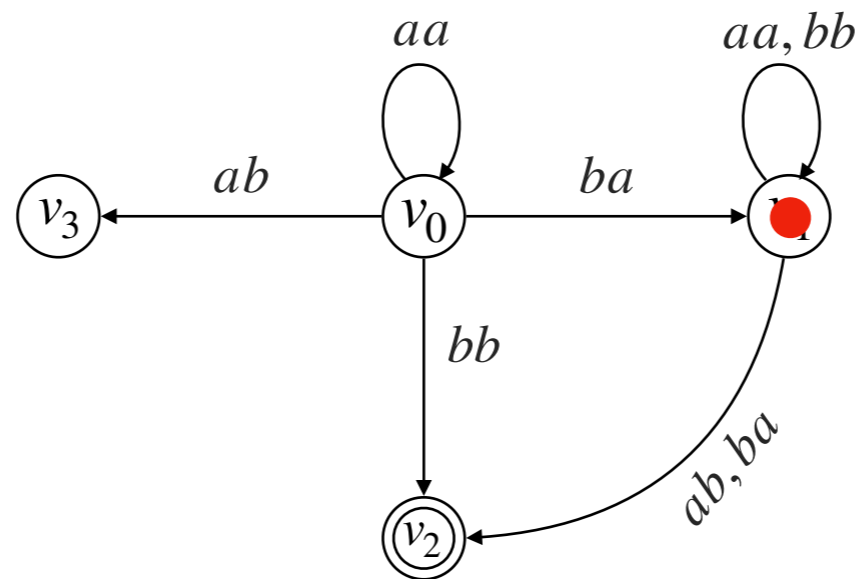
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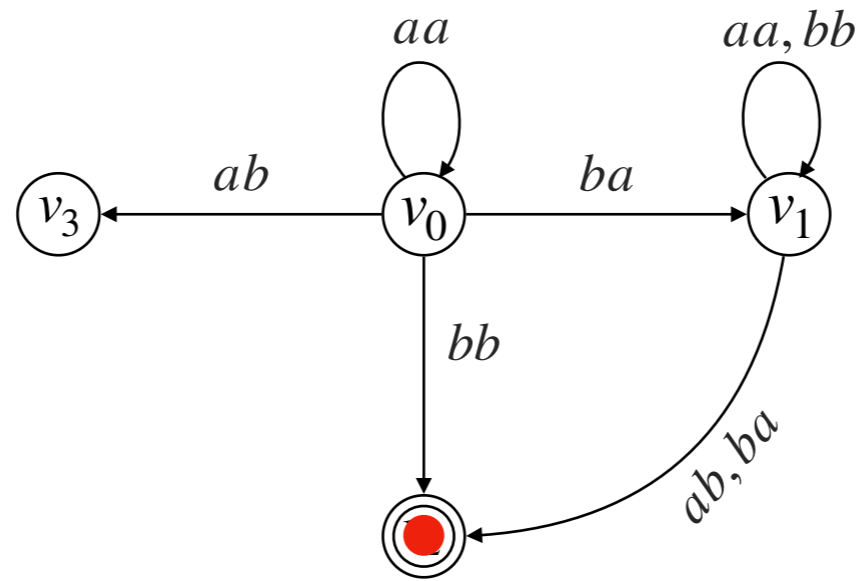
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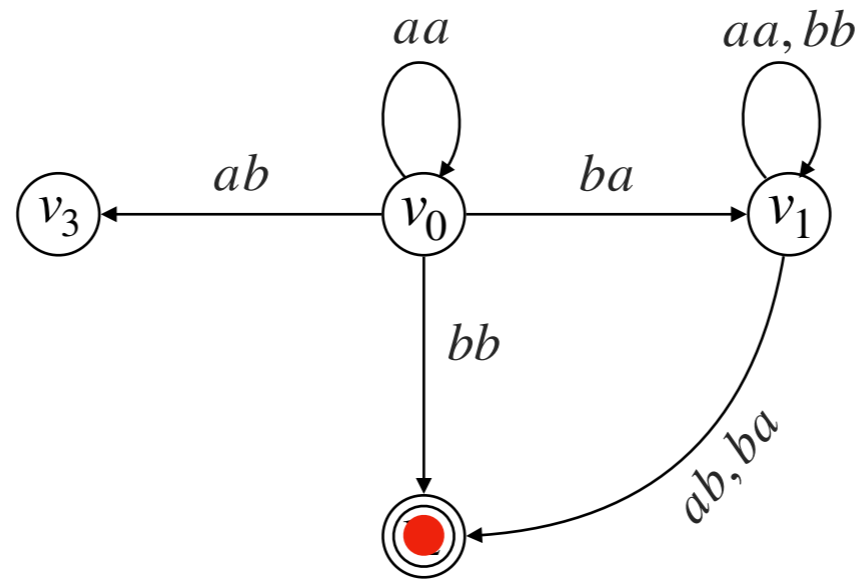
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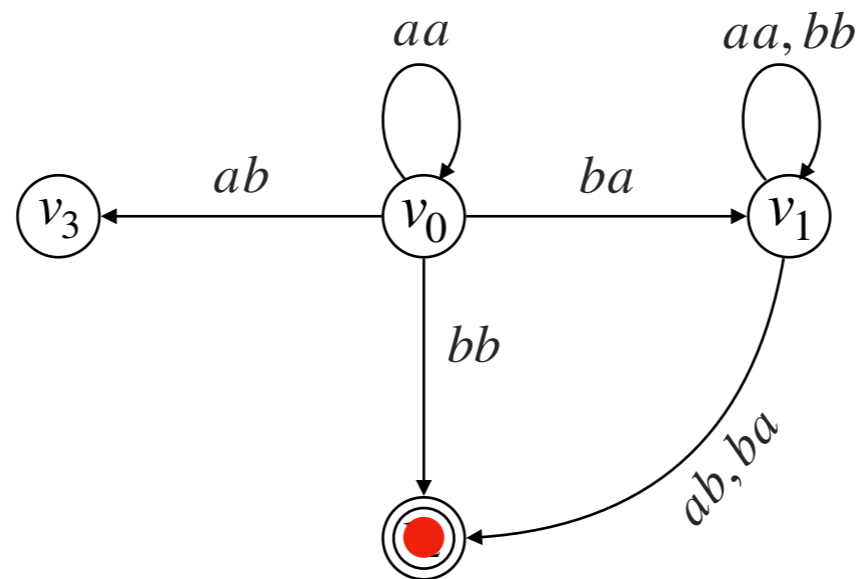
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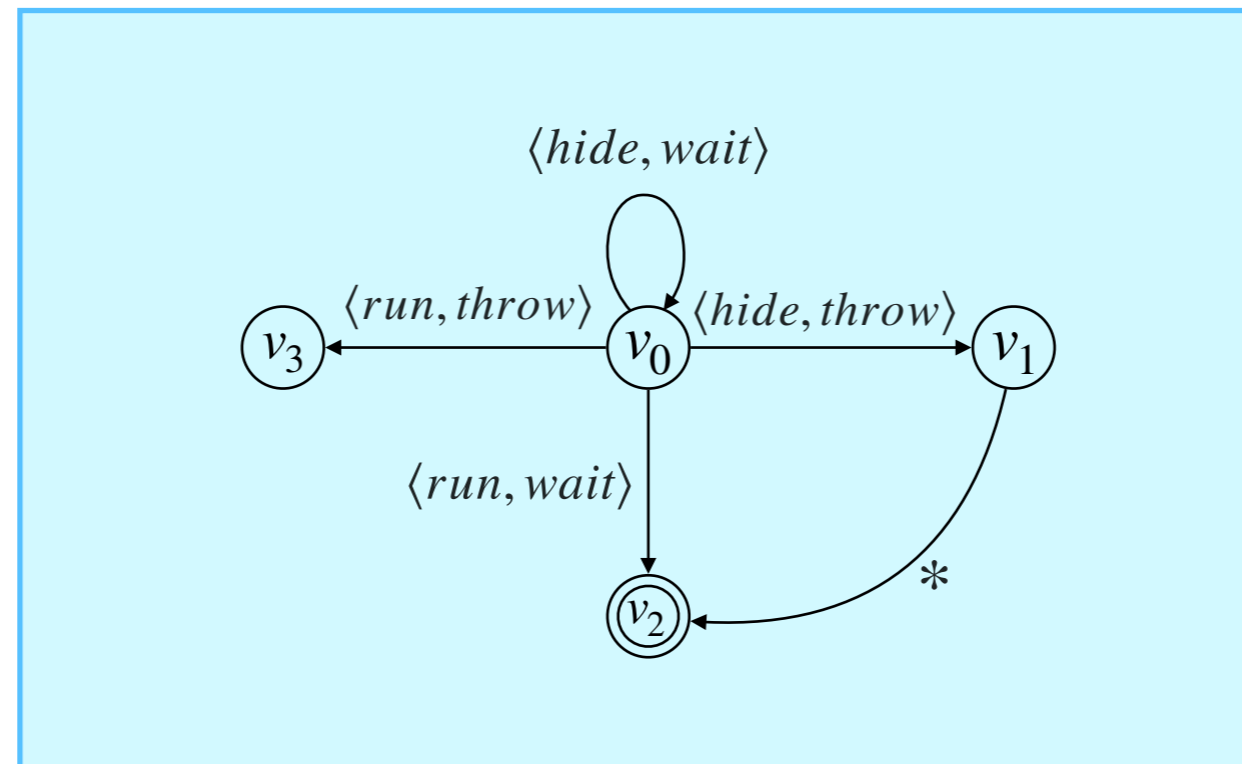
- ▶ Examples of winning objectives: **Reachability, Safety...**

Example

[Alfaro, Henzinger, Kupferman '07]

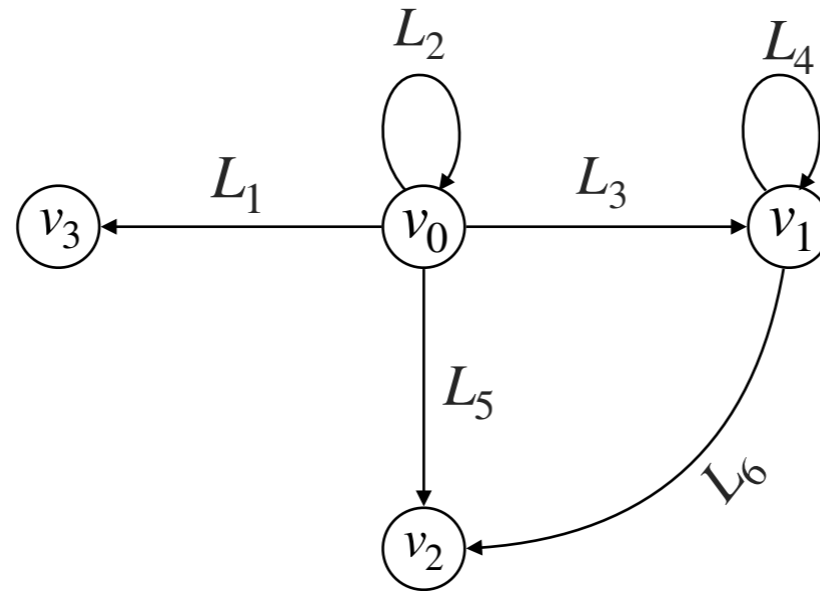
Hide-or-run example

► Player 1 wants to reach home safely when Player 2 wants to throw a snowball at him.



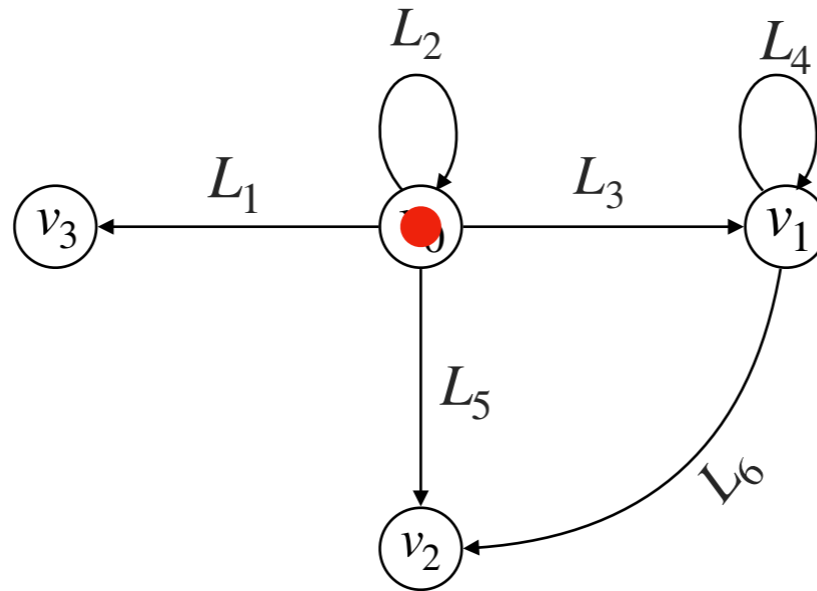
► No player has a winning strategy.

Parameterized concurrent game arena



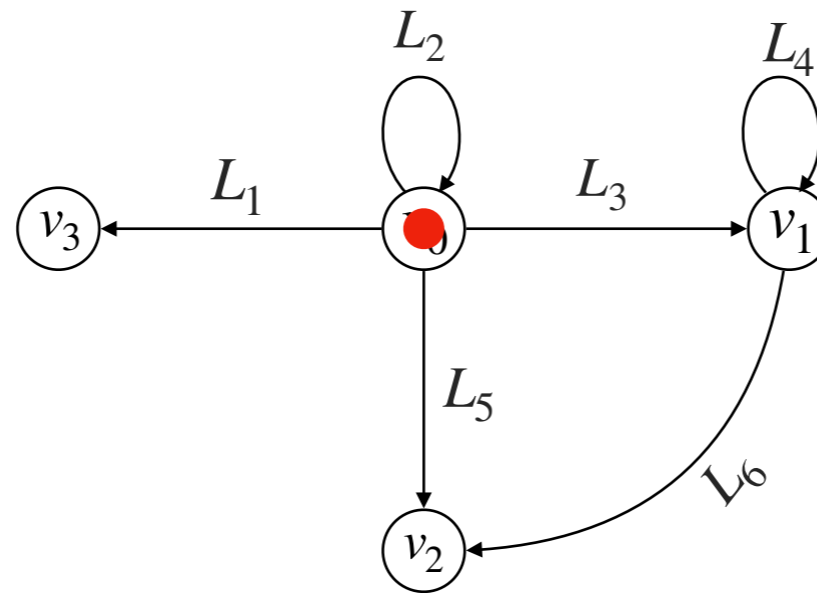
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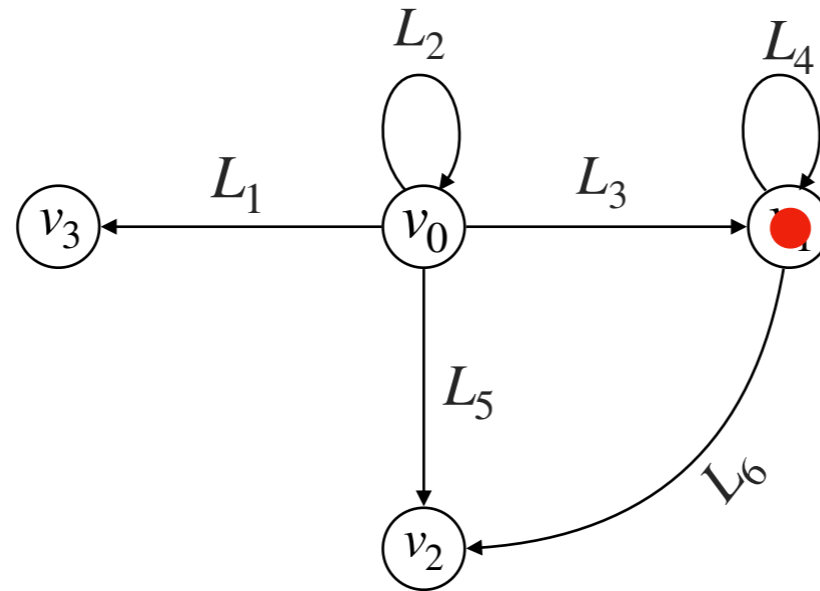
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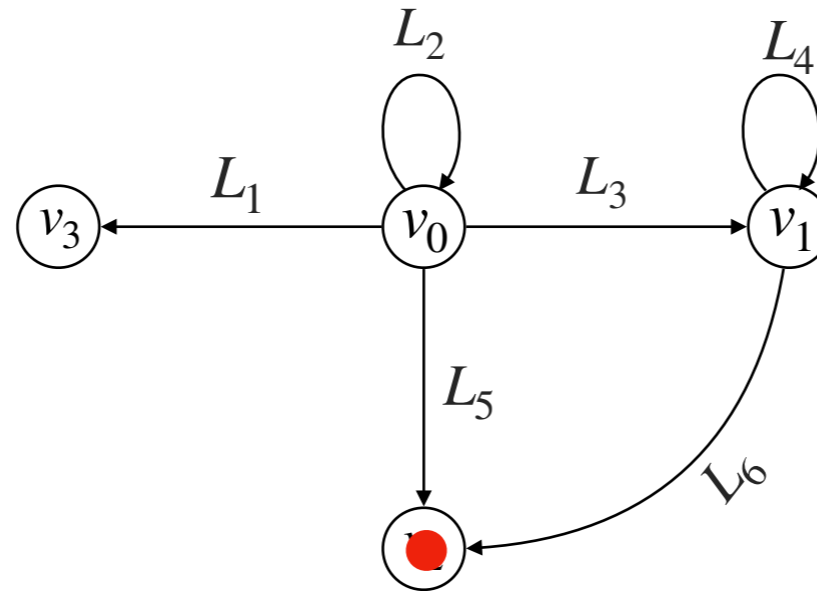
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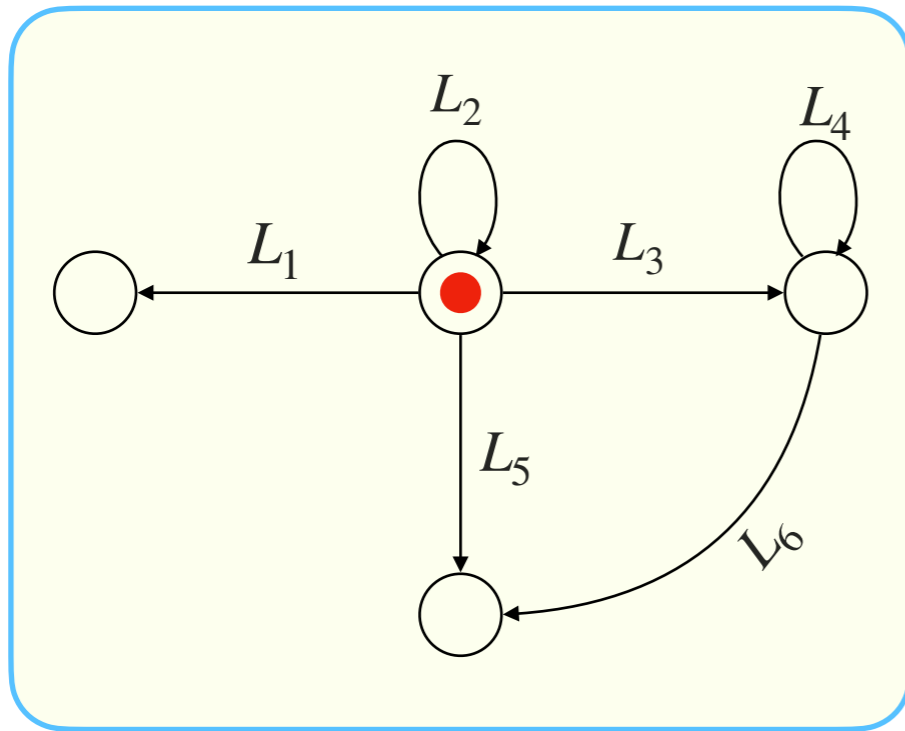
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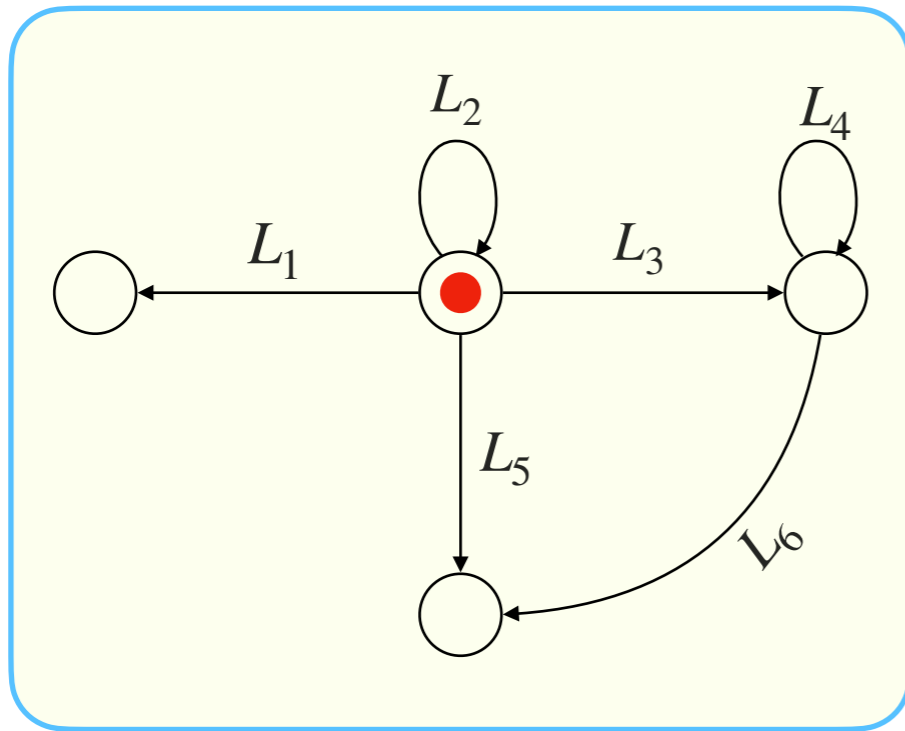
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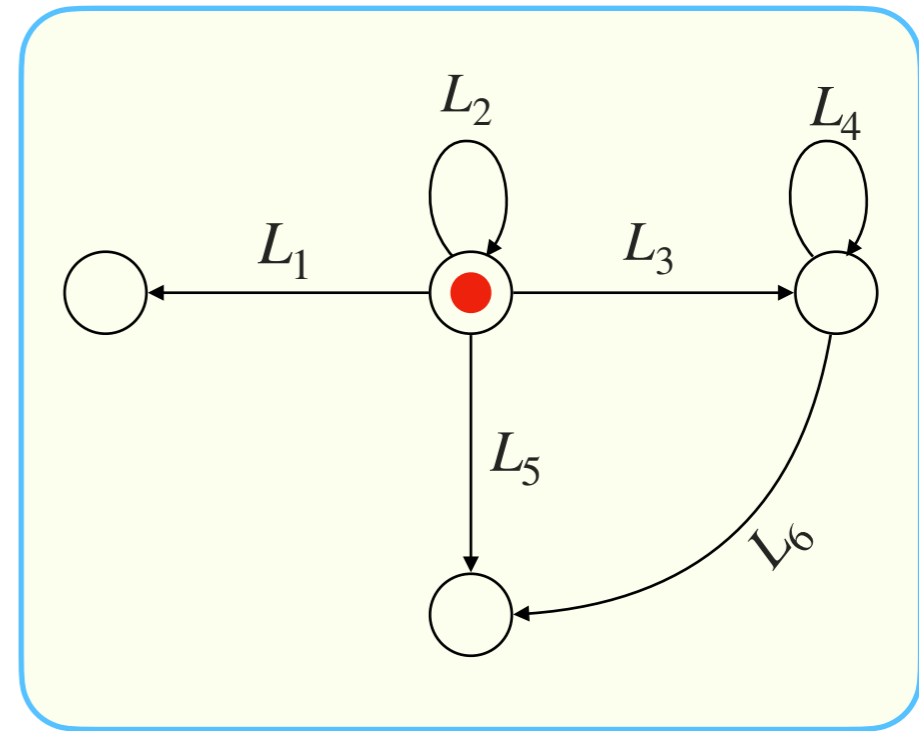


A **distinguished player** trying to achieve a goal against arbitrary number of opponents.

Decision problems

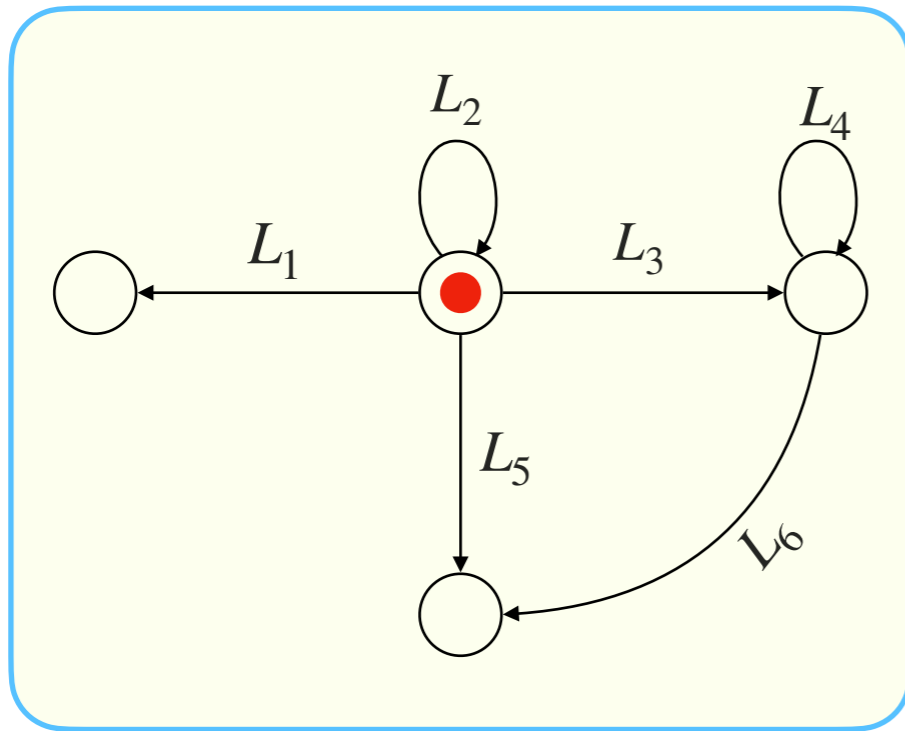


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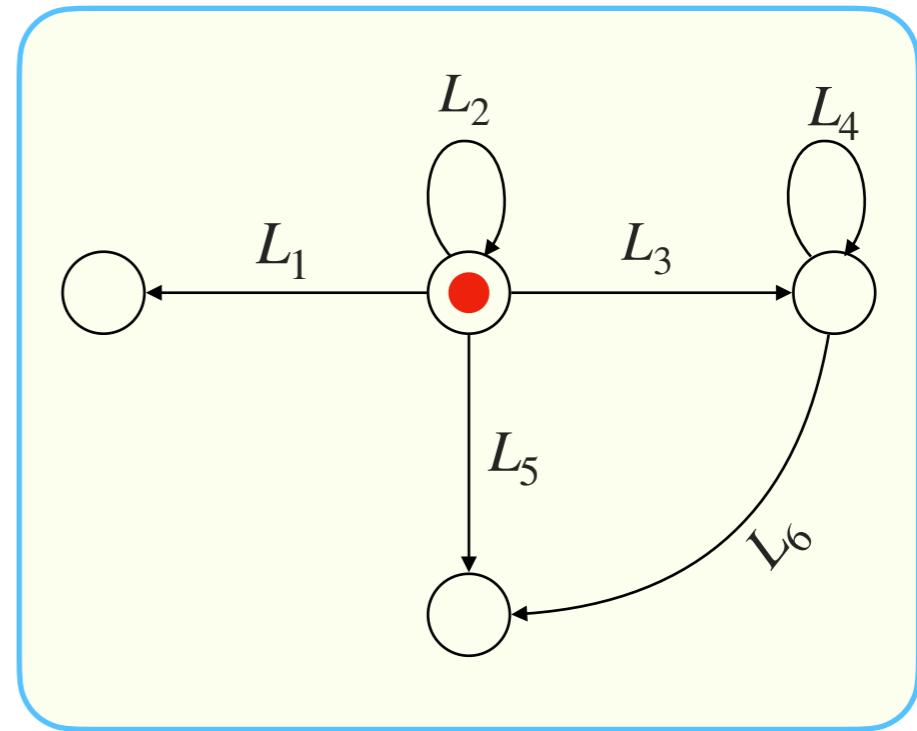
Arbitrary number of players trying to achieve a **common goal**.

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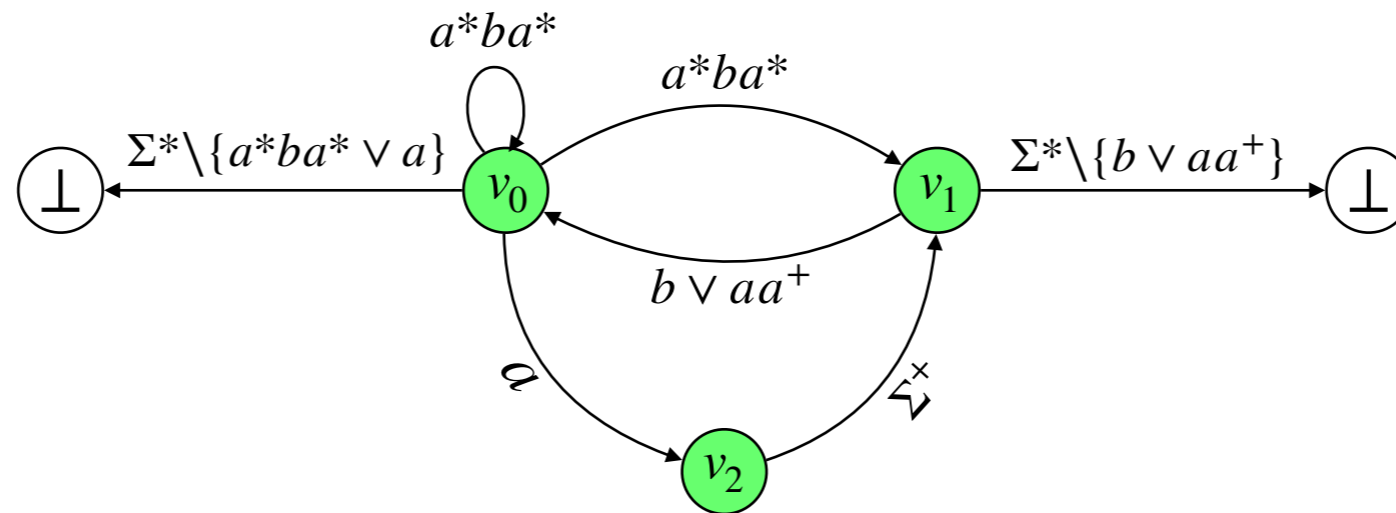
[FSTTCS 2019]



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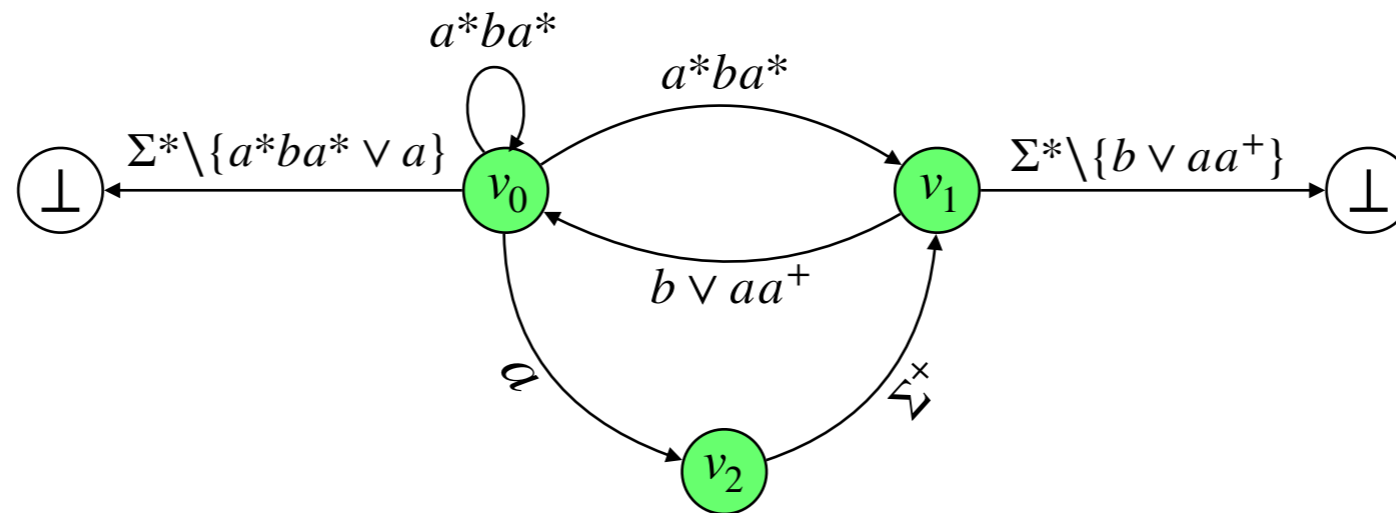
[This work]

Safe coalition problem



► **Strategy** of player i is $\sigma_i : V^+ \rightarrow \Sigma$.

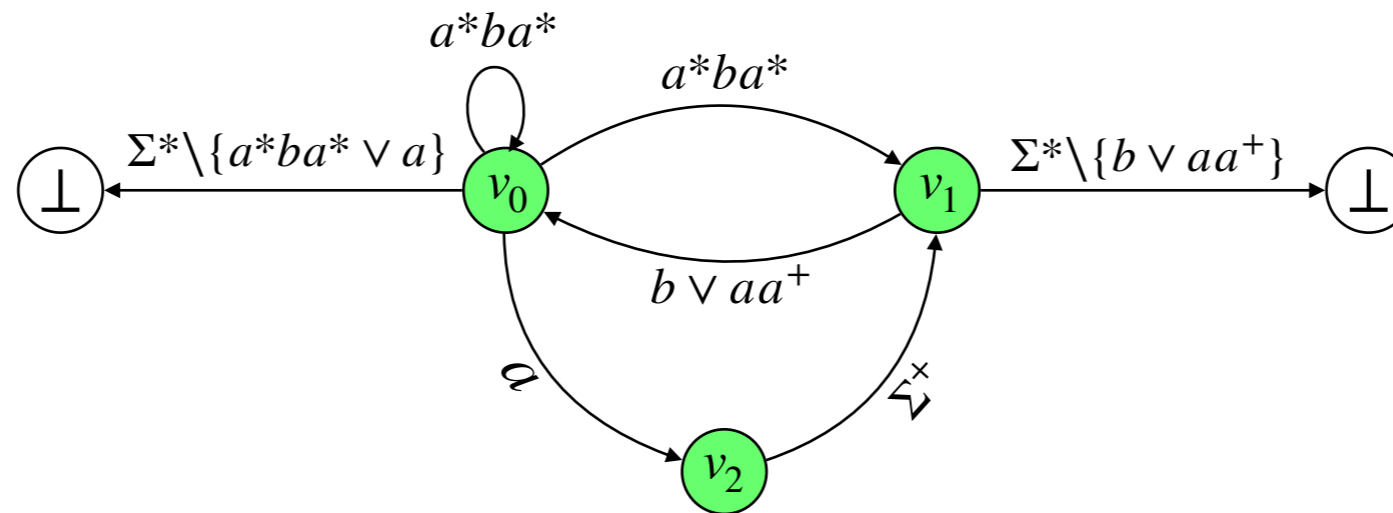
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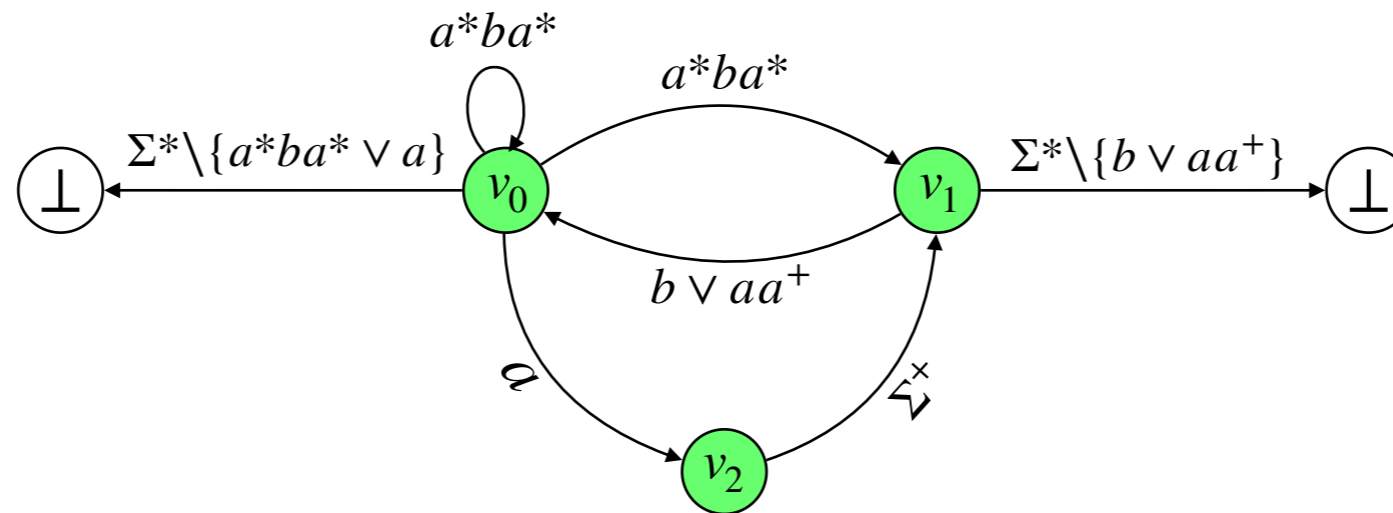
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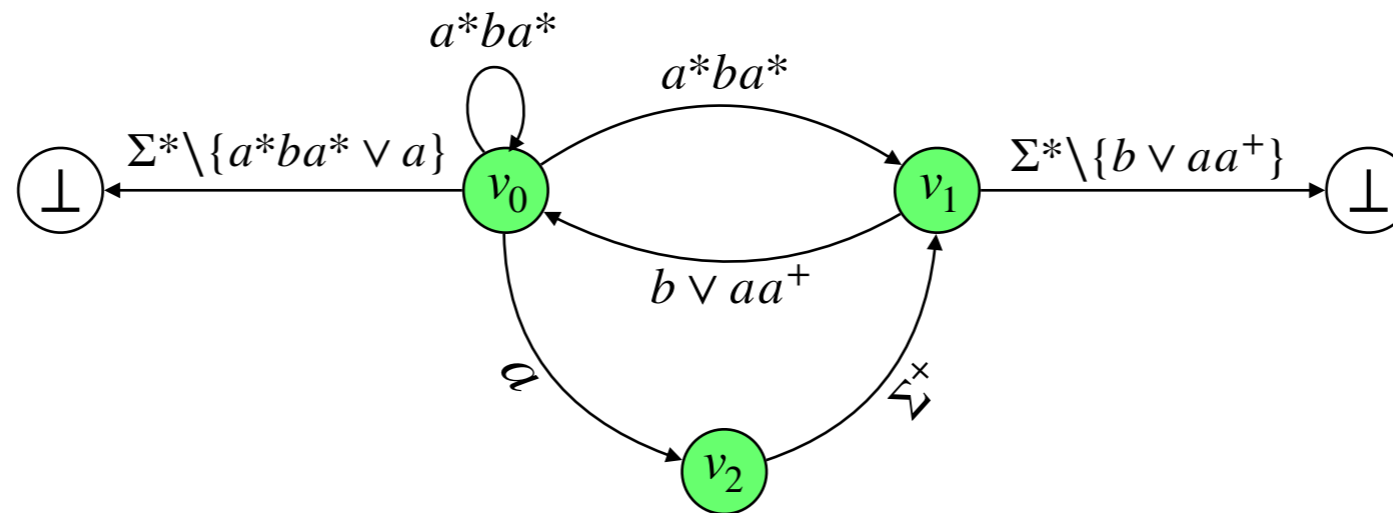
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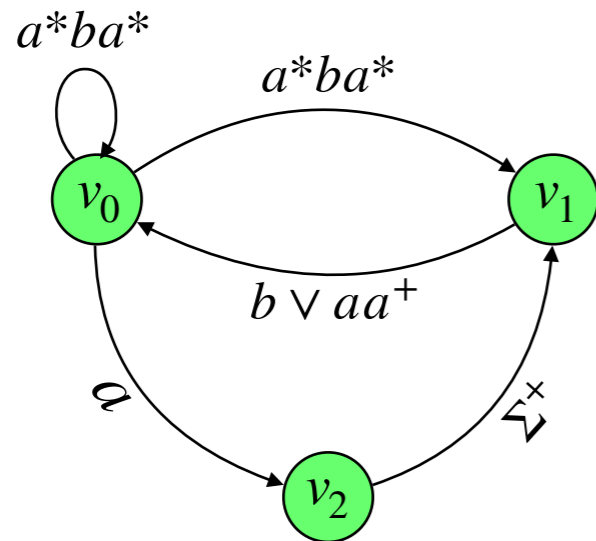
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Input: Arena \mathcal{A} , initial vertex $v_0 \in V$ and set of **safe vertices** S .

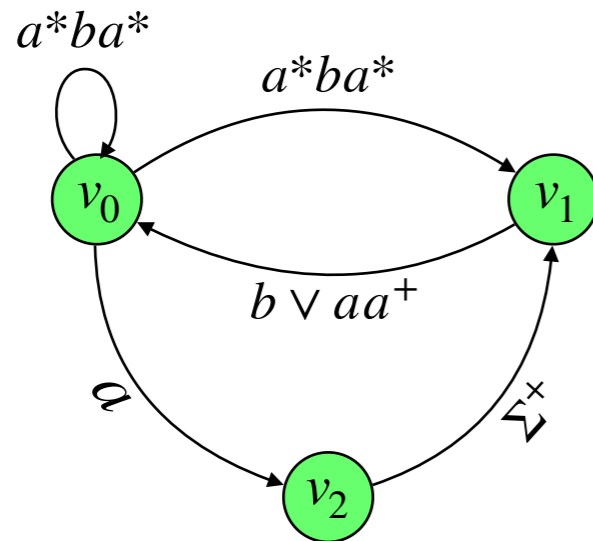
Output: Yes iff $\exists \tilde{\sigma} . \forall k . Out^k(v_0, \tilde{\sigma}) \subseteq S^\omega$.

Example



- $\Sigma = \{a, b\}$.
- Unspecified transitions lead to a **losing** vertex \perp .
- Coalition needs to stay within the **safe** vertices.

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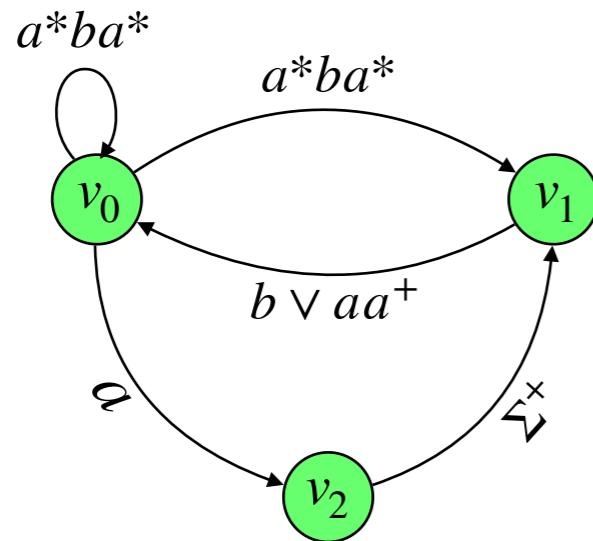


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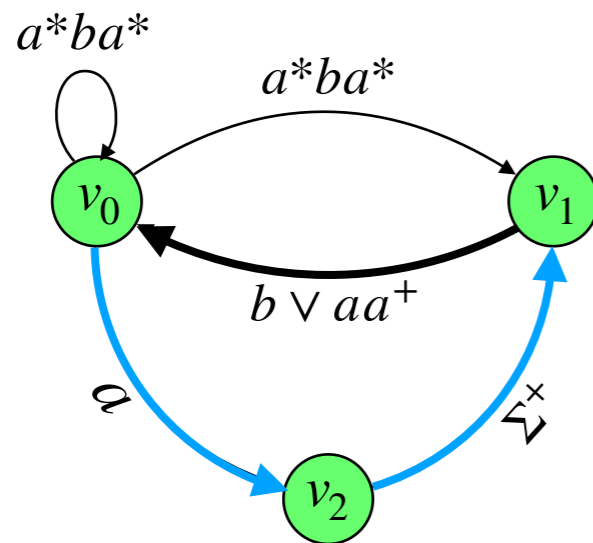
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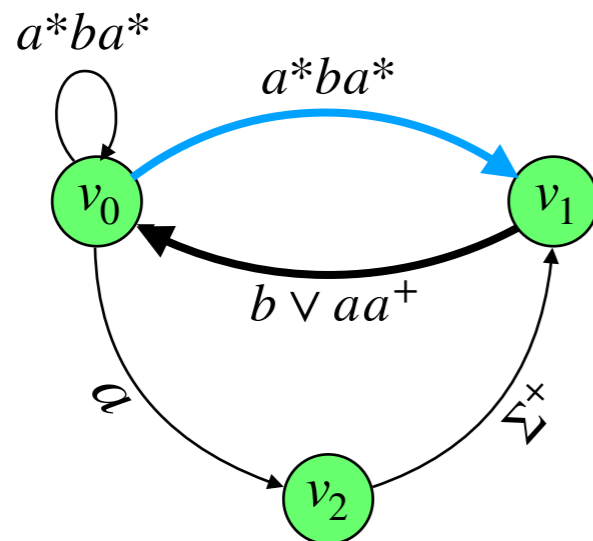
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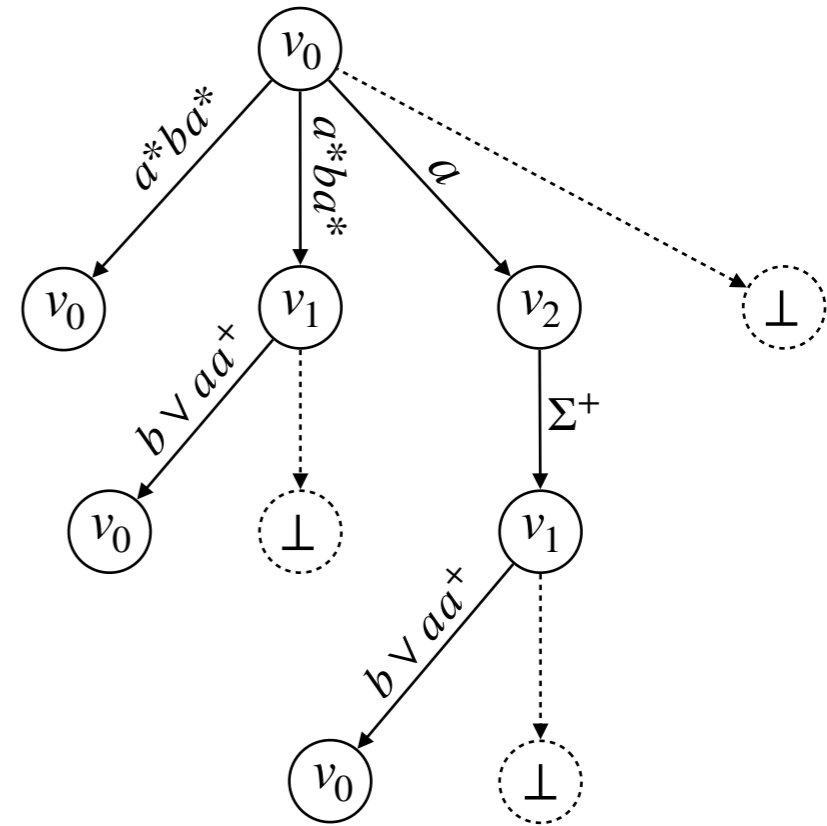
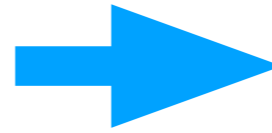
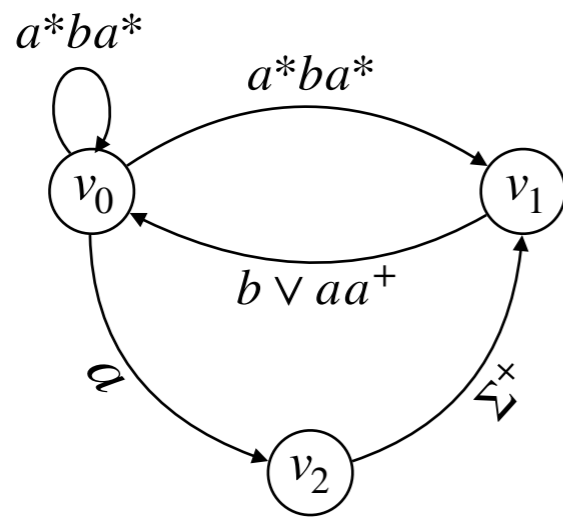
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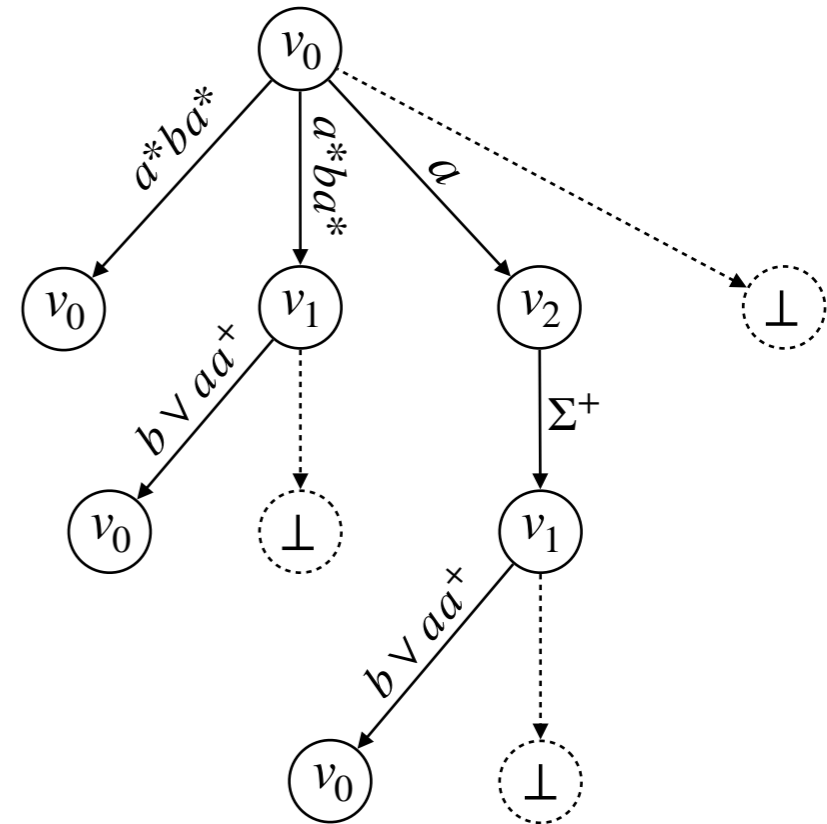
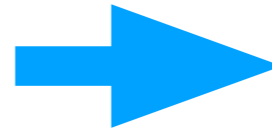
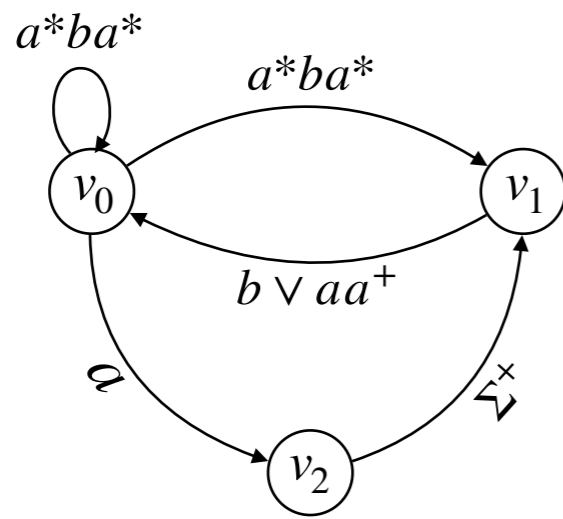
Resolution of safe coalition problem



► Unfold arena \mathcal{A} to a finite **tree**.

🧑 Label nodes with corresponding vertices, and edges with languages.

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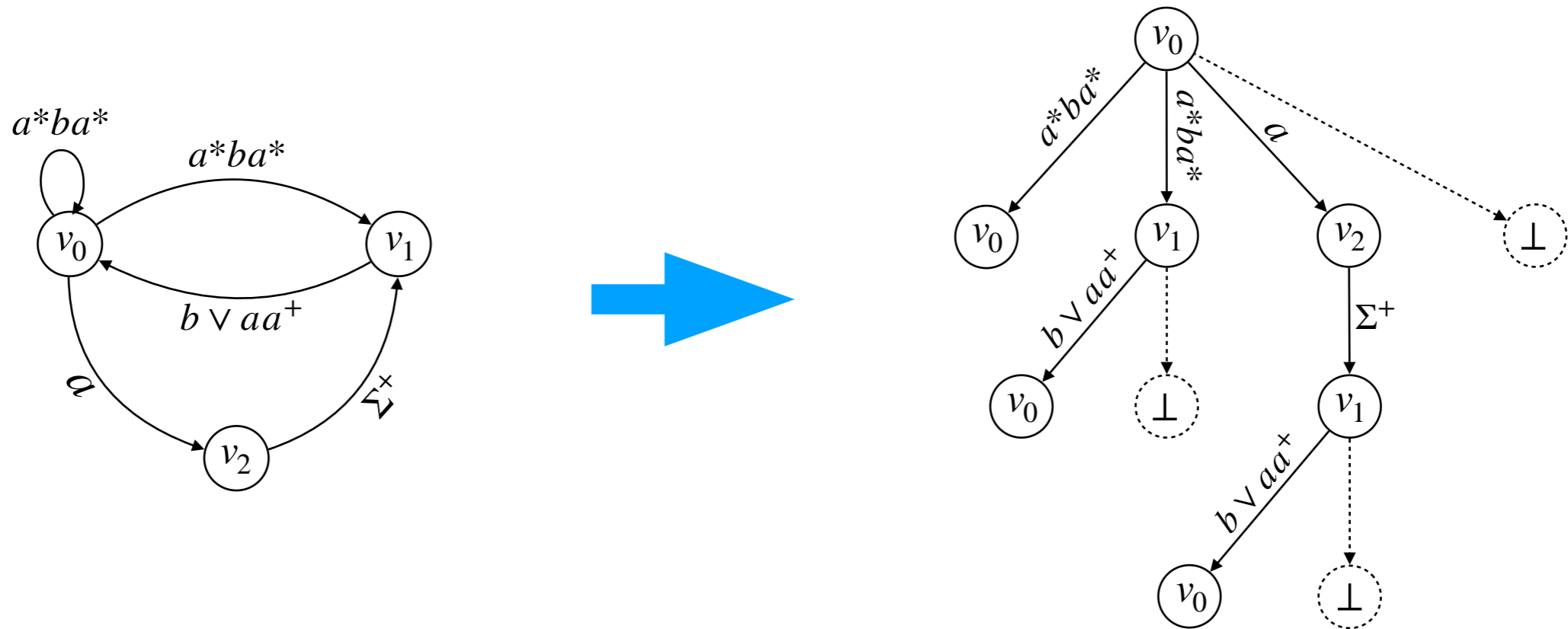
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► Terminate a branch if:

• **either** some label repeats in the same branch,

• **or** the label is not in S .

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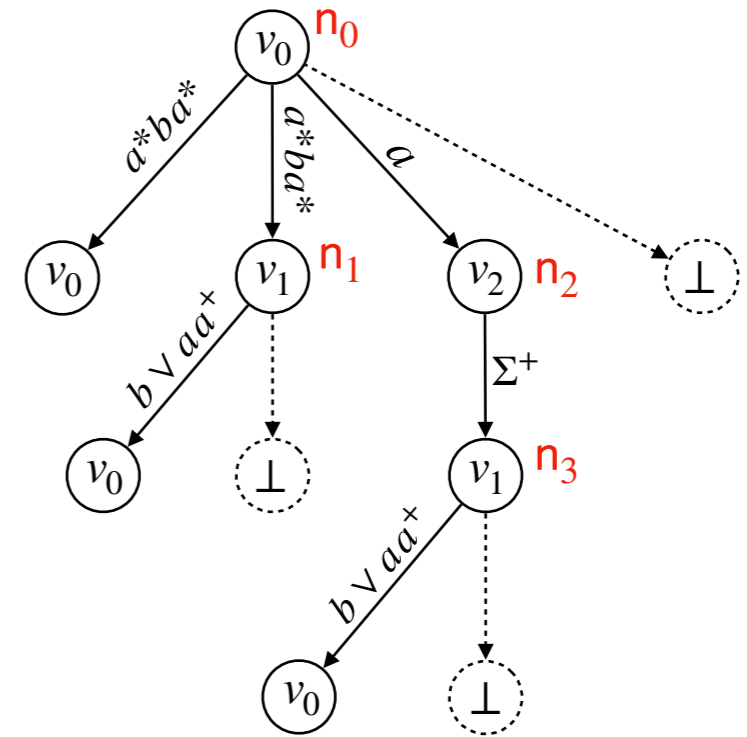
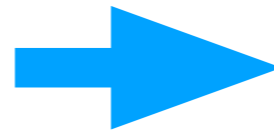
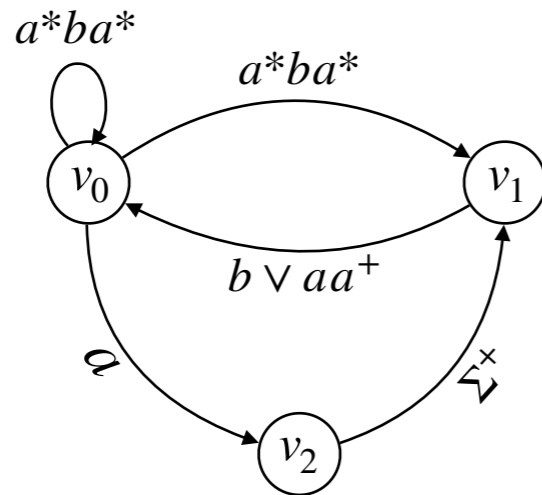
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► Intuitively, if a vertex repeats in \mathcal{A} , coalition may take the same strategy.

• If it ensures **safety** in the first occurrence, then also for the later.

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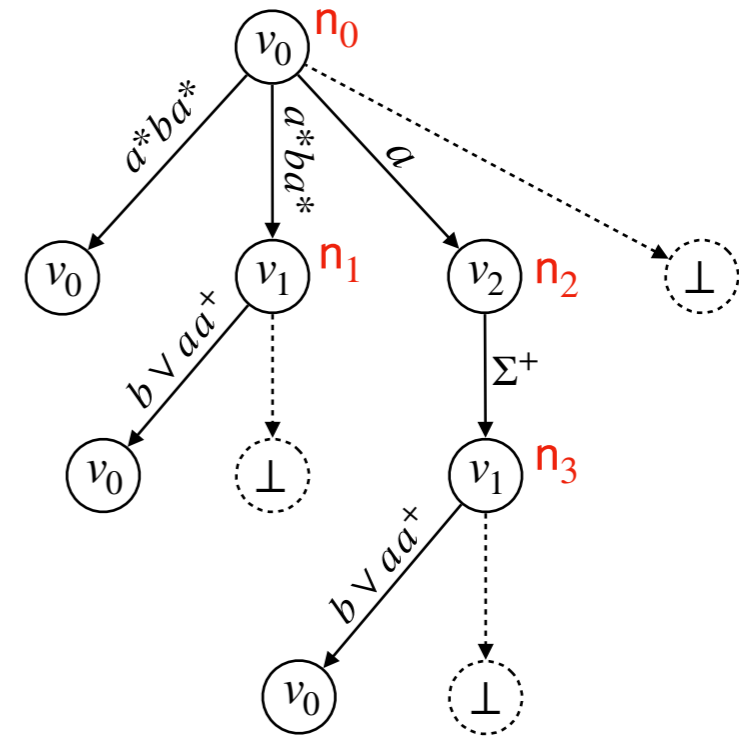
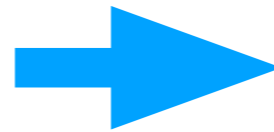
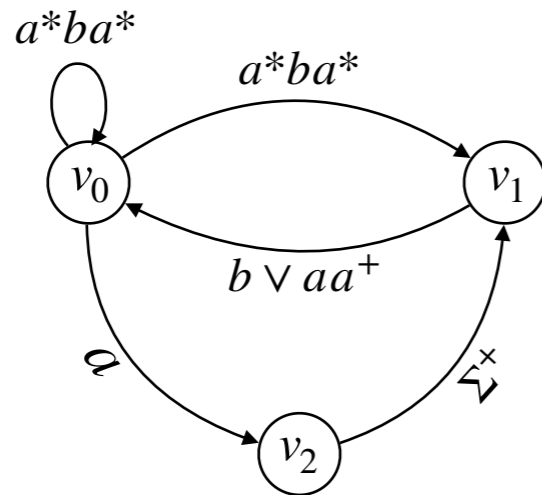
Correctness of Tree Unfolding



► A **coalition Strategy** in the tree is a mapping $\tau : N_{int} \rightarrow \Sigma^\omega$.

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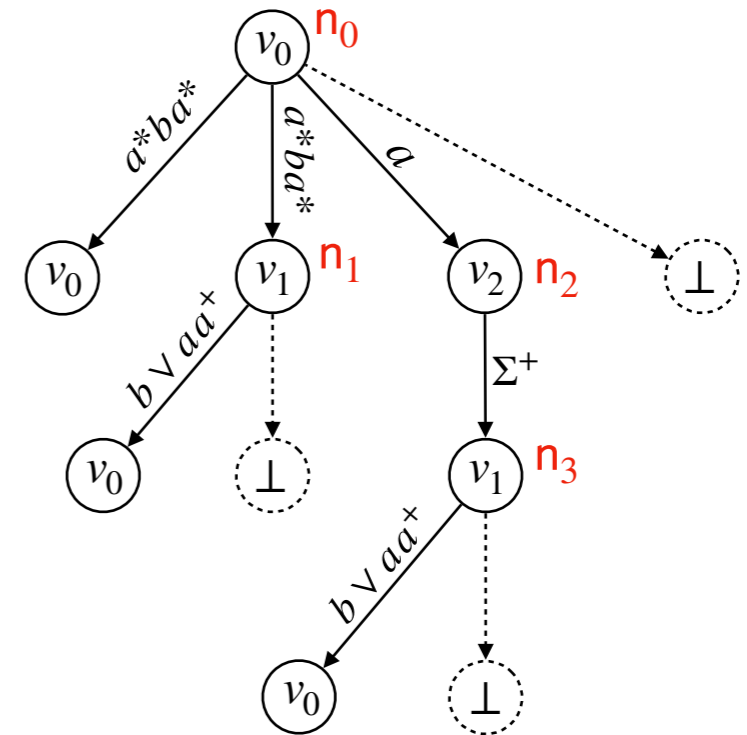
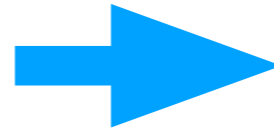
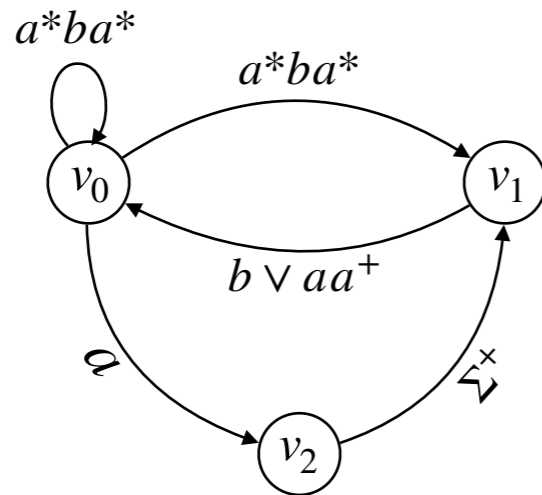


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The coalition wins if they can reach a **safe** leaf.

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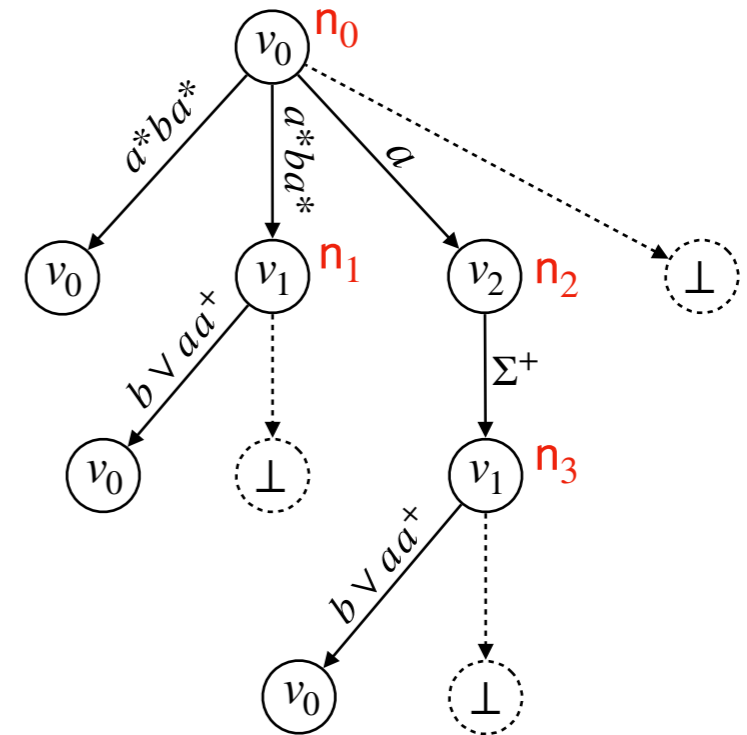
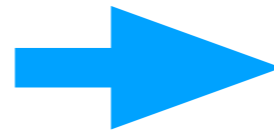
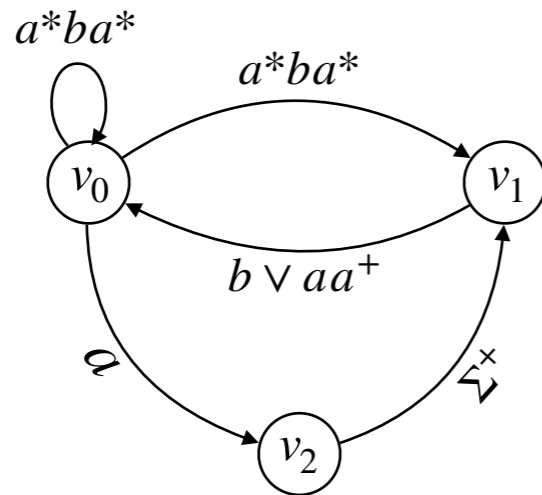


Coalition has a winning strategy in \mathcal{T}

📌 Proof idea: any history V^+ in \mathcal{A} **uniquely** maps to an internal node in \mathcal{T} .

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Coalition has a winning strategy in \mathcal{T}

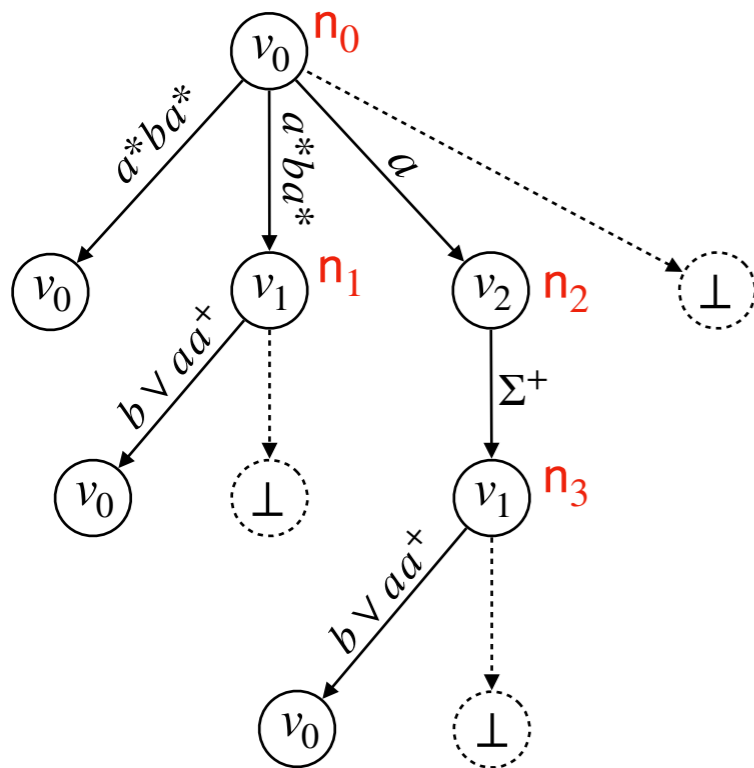
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Safe coalition problem **reduces** to existence of a winning coalition strategy in the finite tree unfolding.

Decidability of safe coalition problem

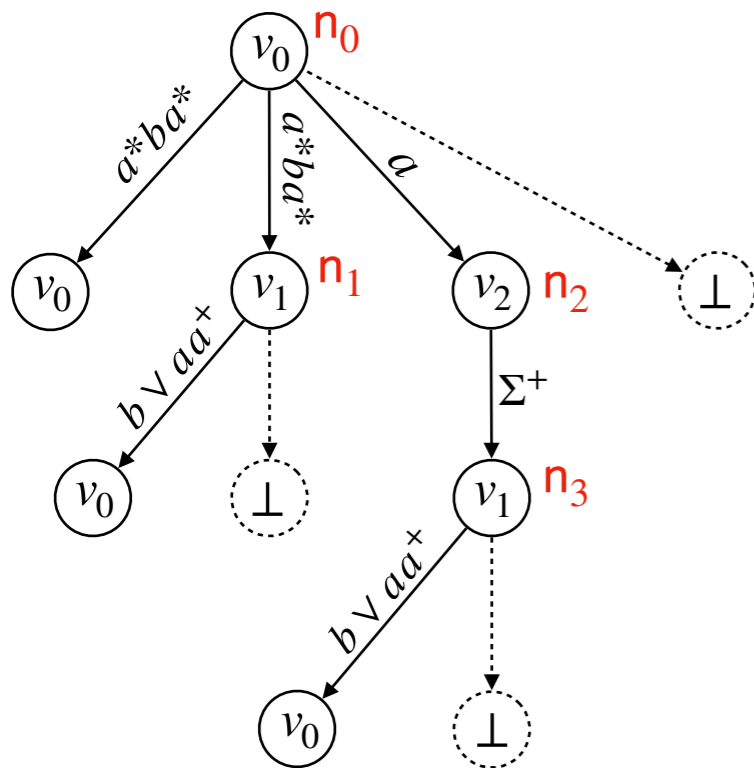
EXPSPACE algorithm

- ▶ m = number of internal nodes in \mathcal{T} ; $m = O(2^{|V|})$.
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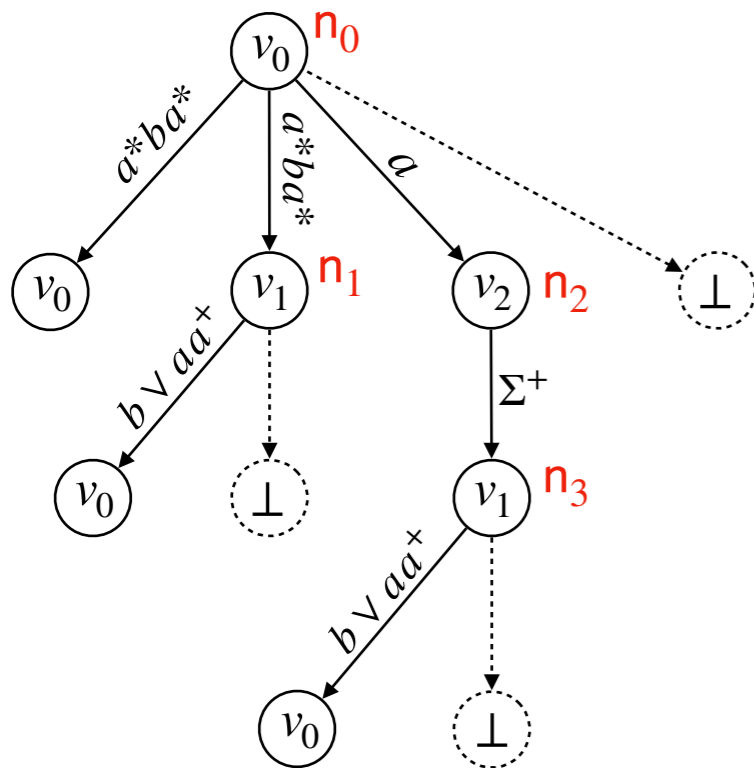
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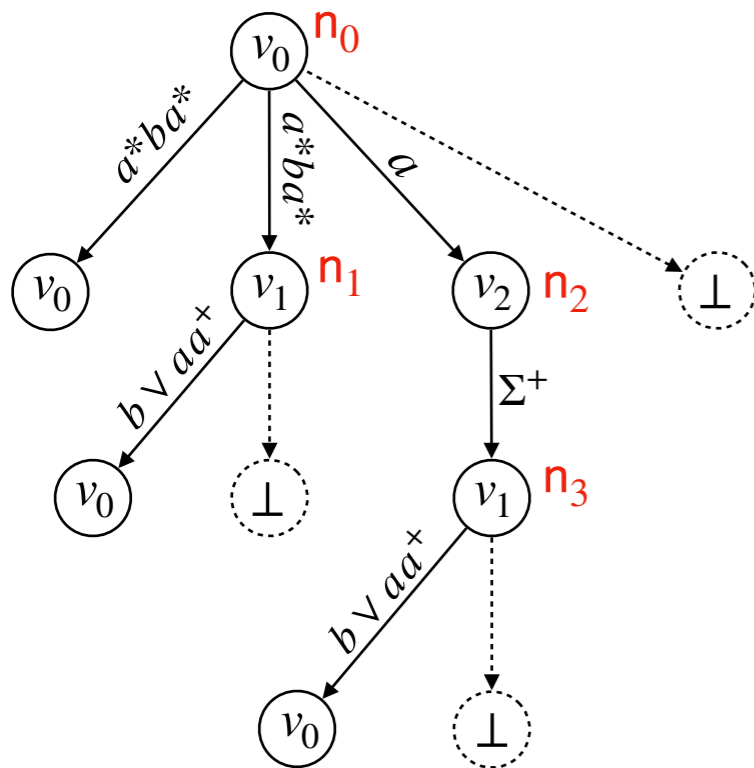
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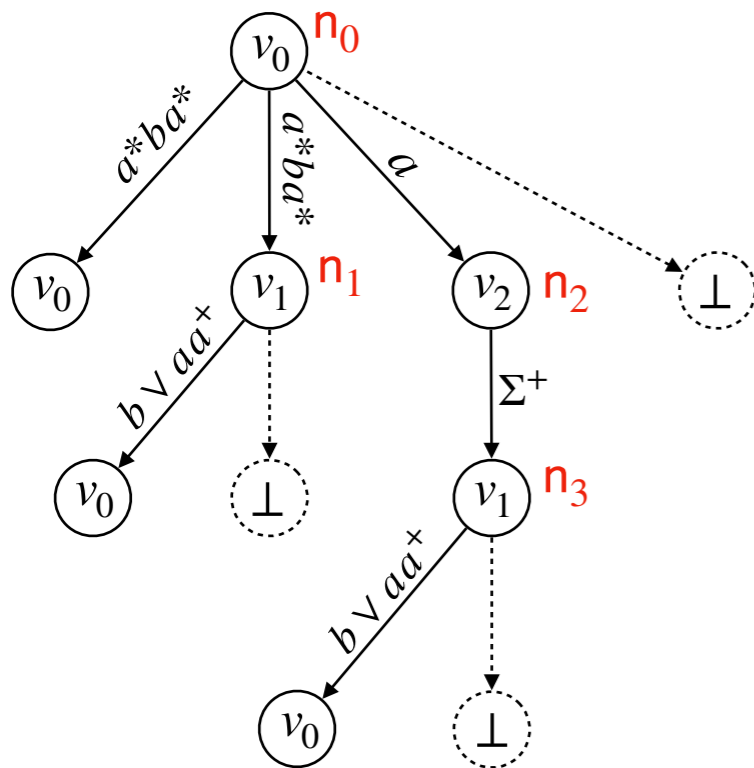
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- ▶ A **coalition Strategy** in \mathcal{T} is a mapping $\tau : N_{int} \rightarrow \Sigma^\omega$.
 - Equivalently, $\tau \in (\Sigma^\omega)^m$.
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- ▶ Construct **safety** automaton \mathcal{B} over alphabet Σ^m :
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Decidability of safe coalition problem

EXPSPACE algorithm



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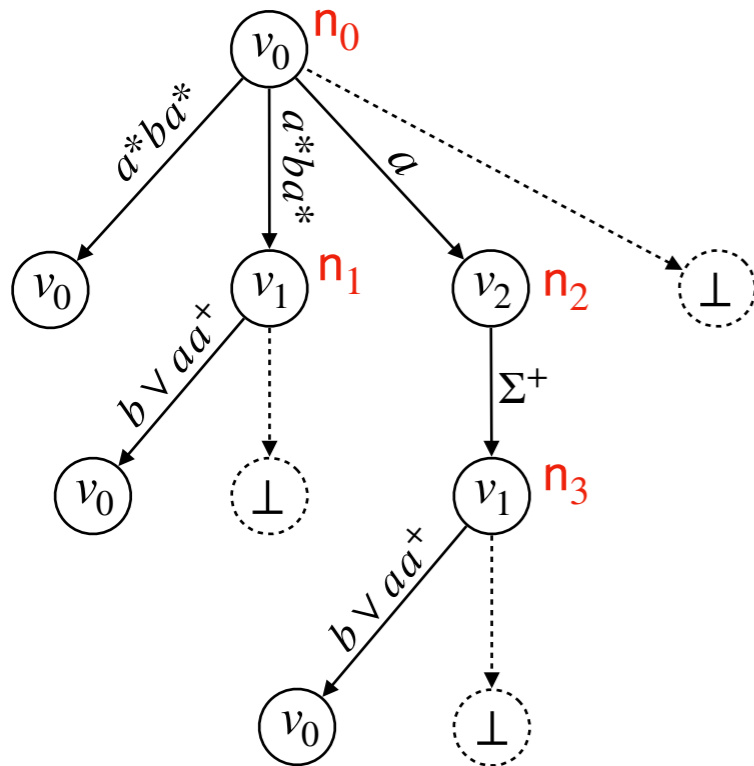
• runs automata on the edges in parallel.

• a (global) state is an r tuple of (local) states.

• a (global) state corresponds to **different** branches.

Decidability of safe coalition problem

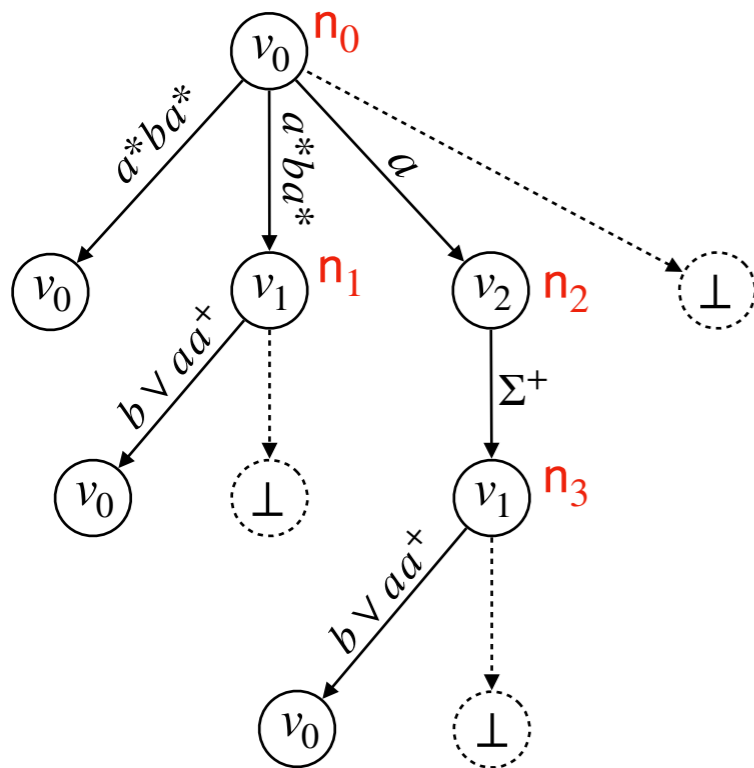
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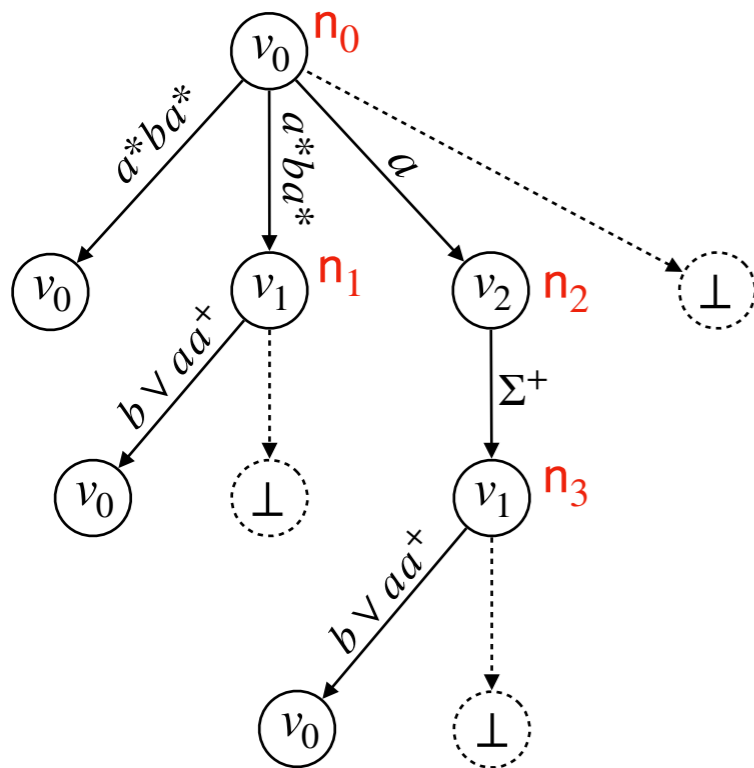
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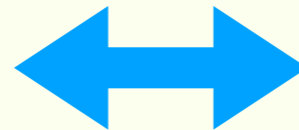
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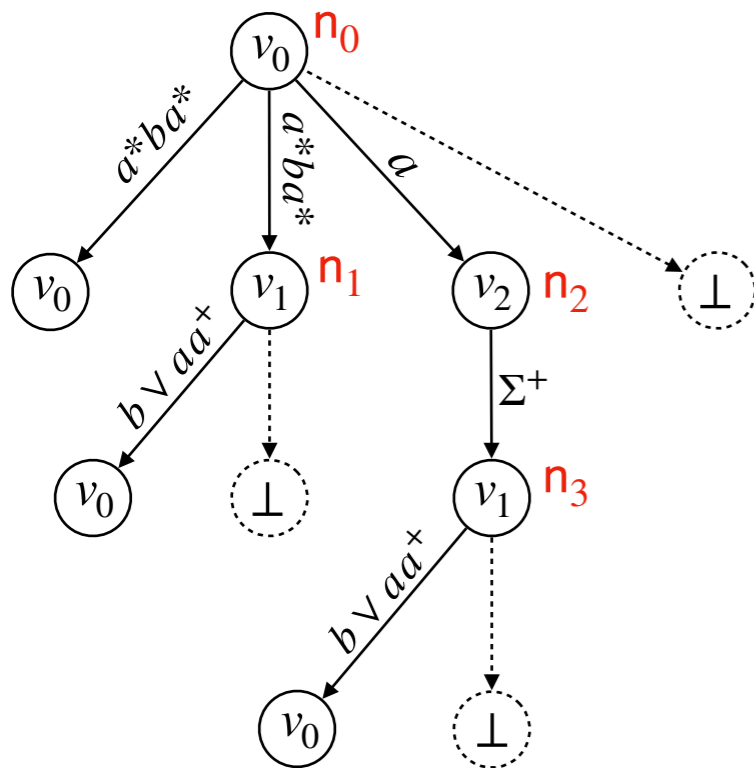
Coalition has a winning strategy in \mathcal{T}



$\mathcal{L}(\mathcal{B}) \neq \emptyset$

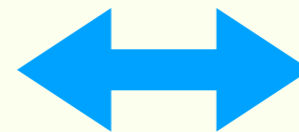
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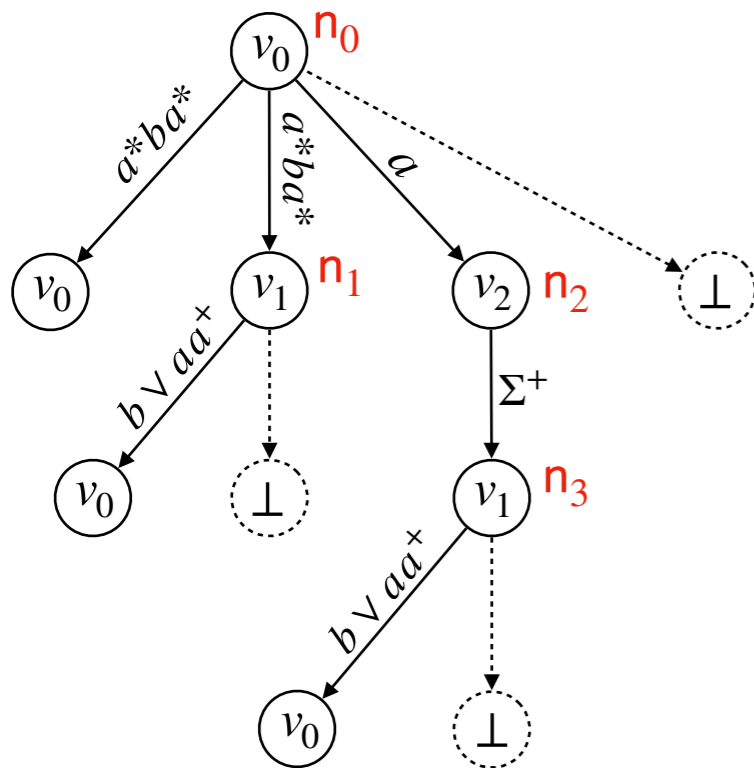


$\mathcal{L}(\mathcal{B}) \neq \emptyset$

• $|\mathcal{B}| = O(2^{2^{|V|}})$.

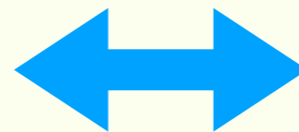
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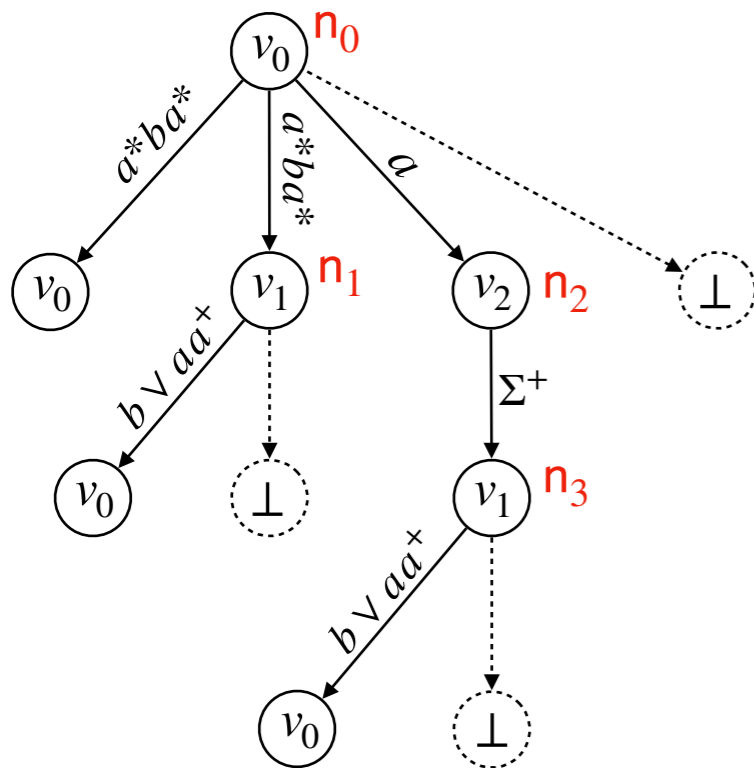
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Safe coalition problem is in **EXPSPACE**

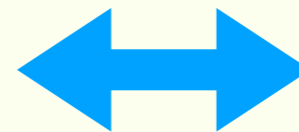
Decidability of safe coalition problem

EXPSpace algorithm



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Coalition has a winning strategy in \mathcal{T}

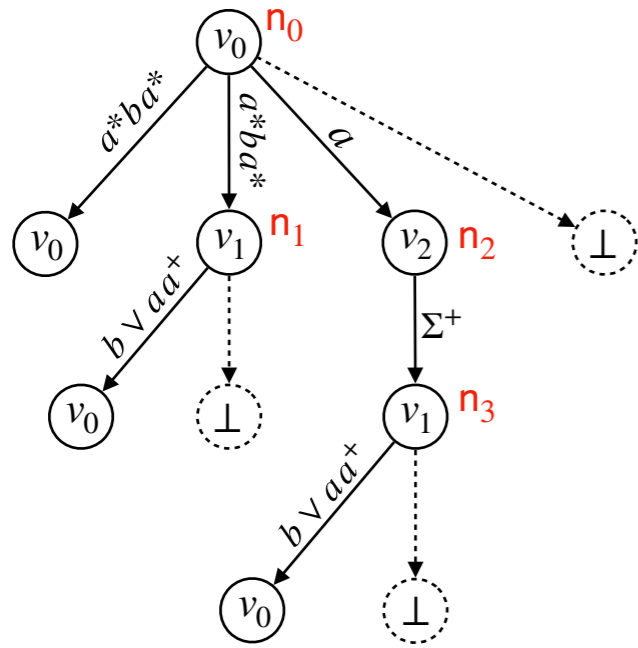


$\mathcal{L}(\mathcal{B}) \neq \emptyset$

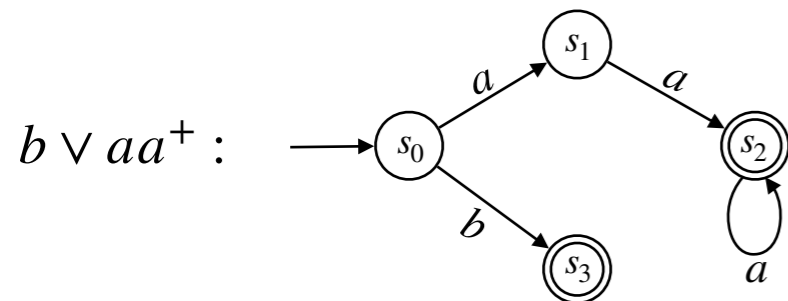
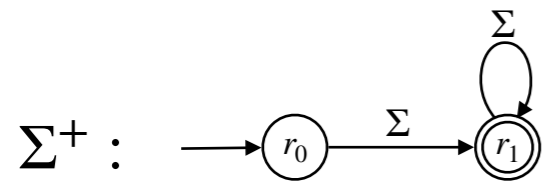
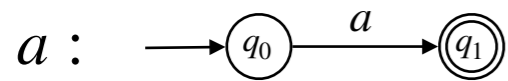
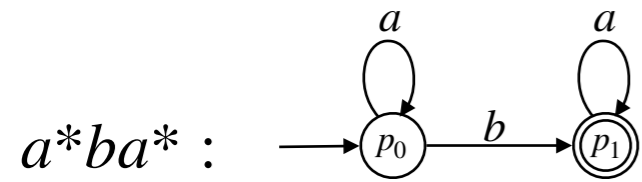
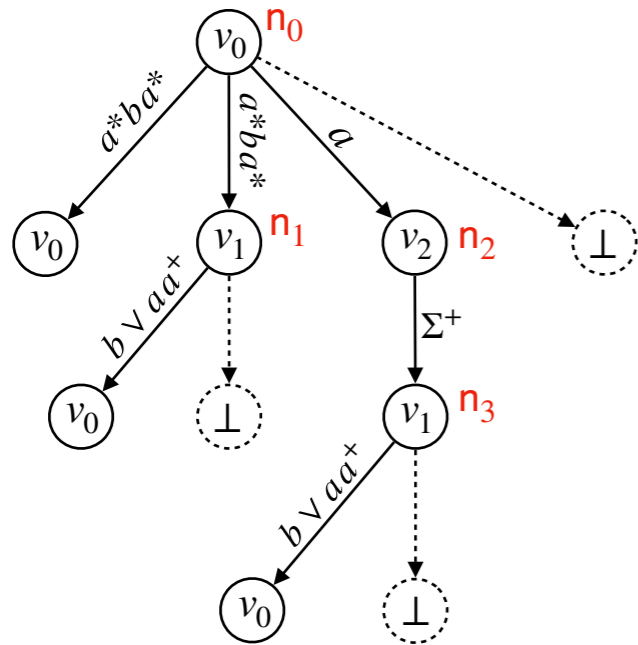
• $|\mathcal{B}| = O(2^{2^{|V|}})$.

Safe coalition problem is in **EXPSpace** and PSPACE-hard.

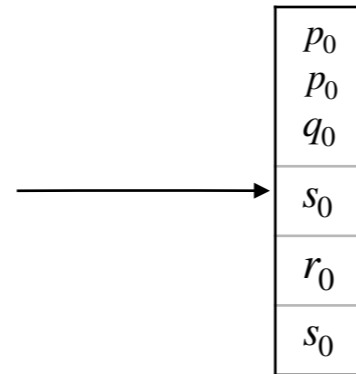
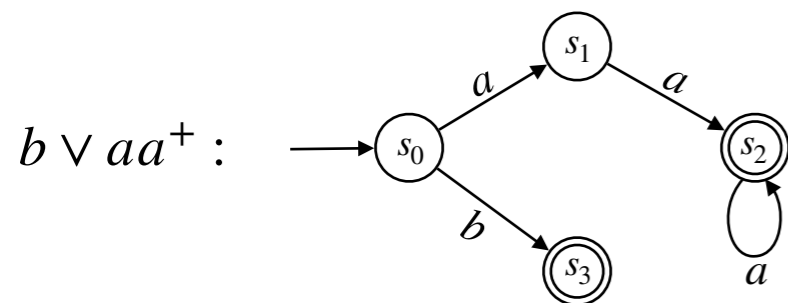
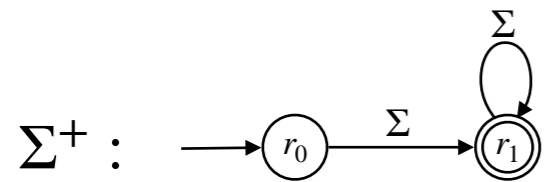
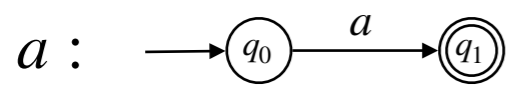
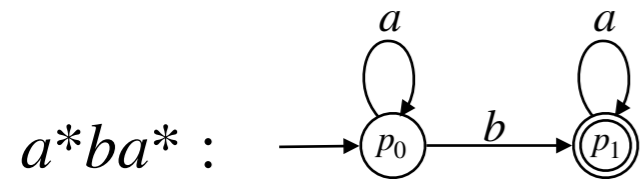
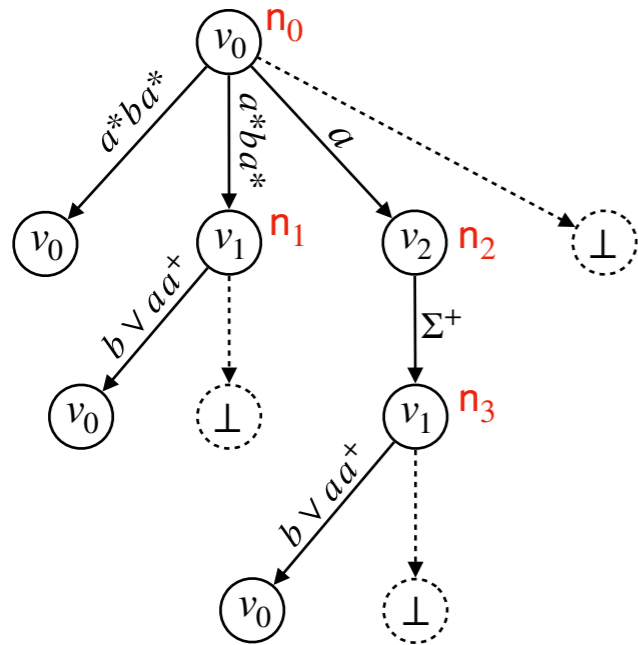
Example



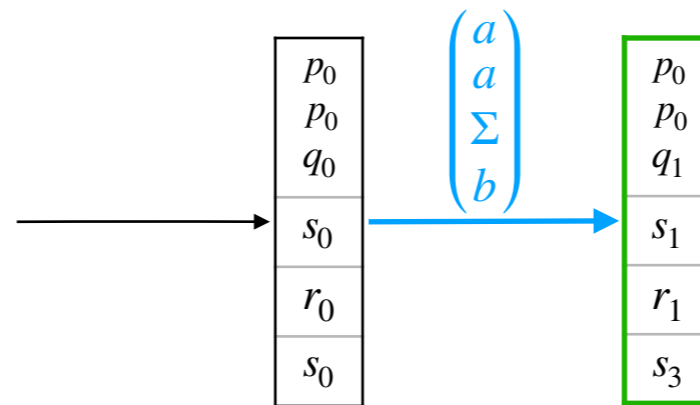
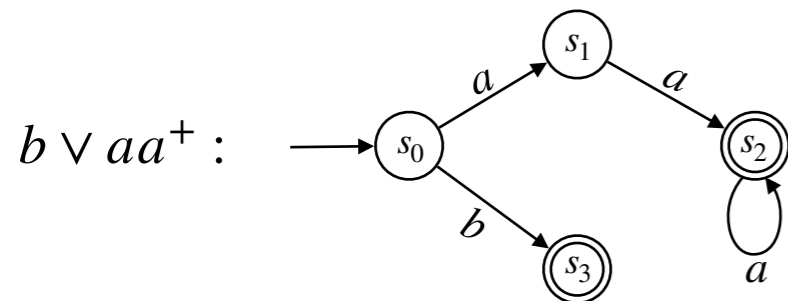
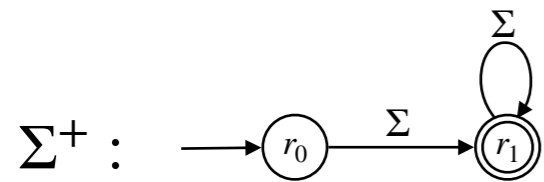
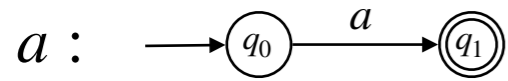
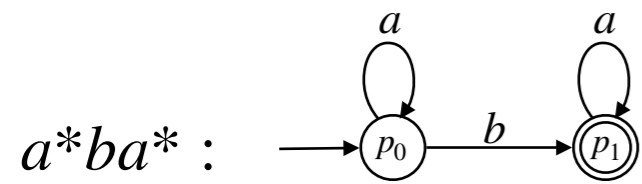
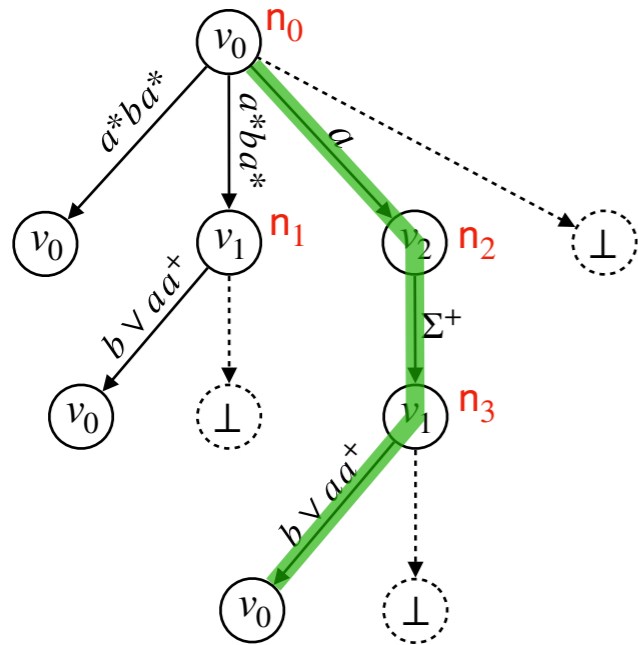
Example



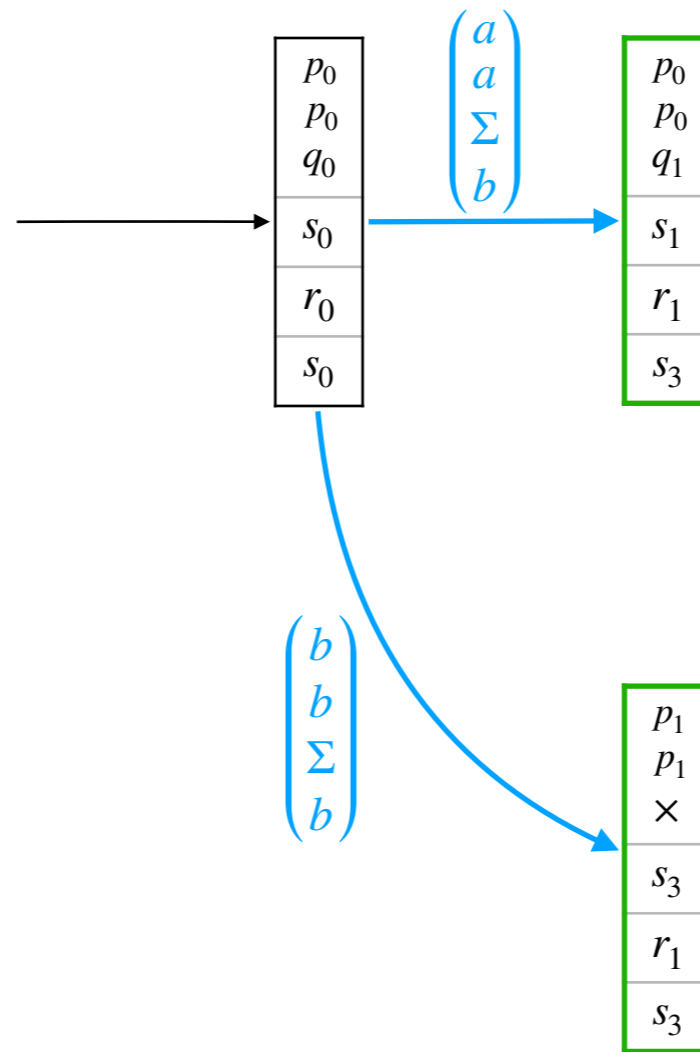
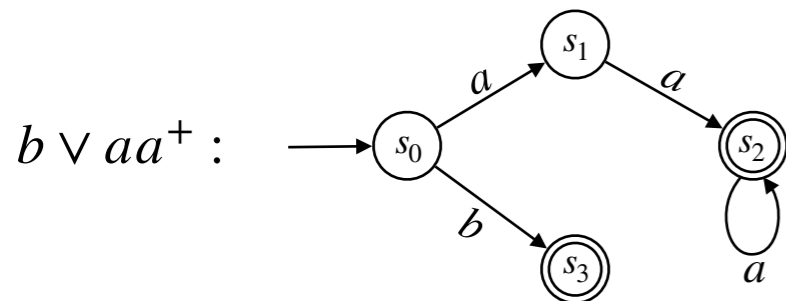
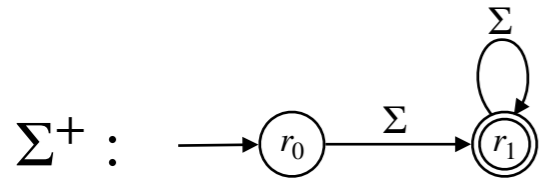
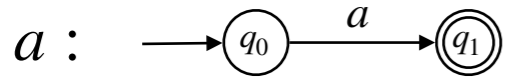
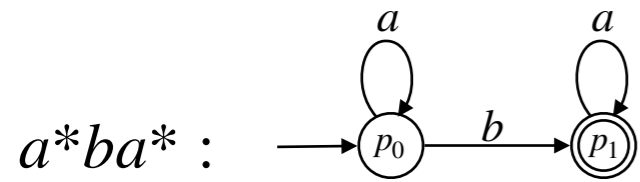
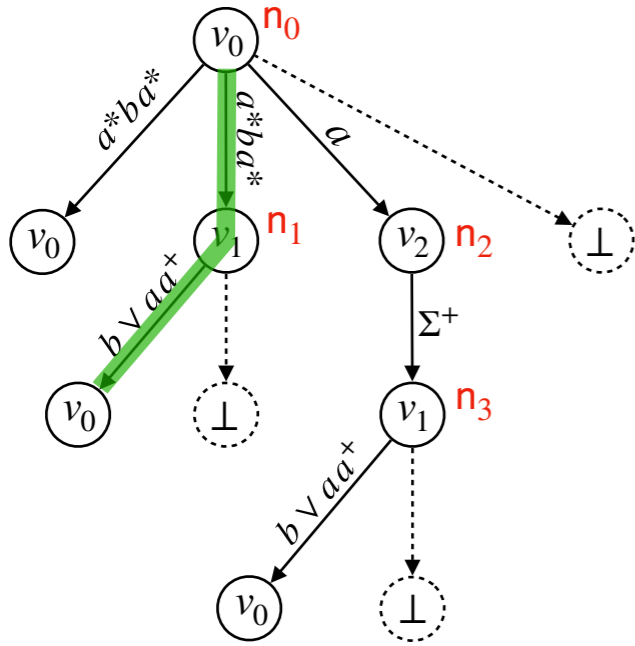
Example



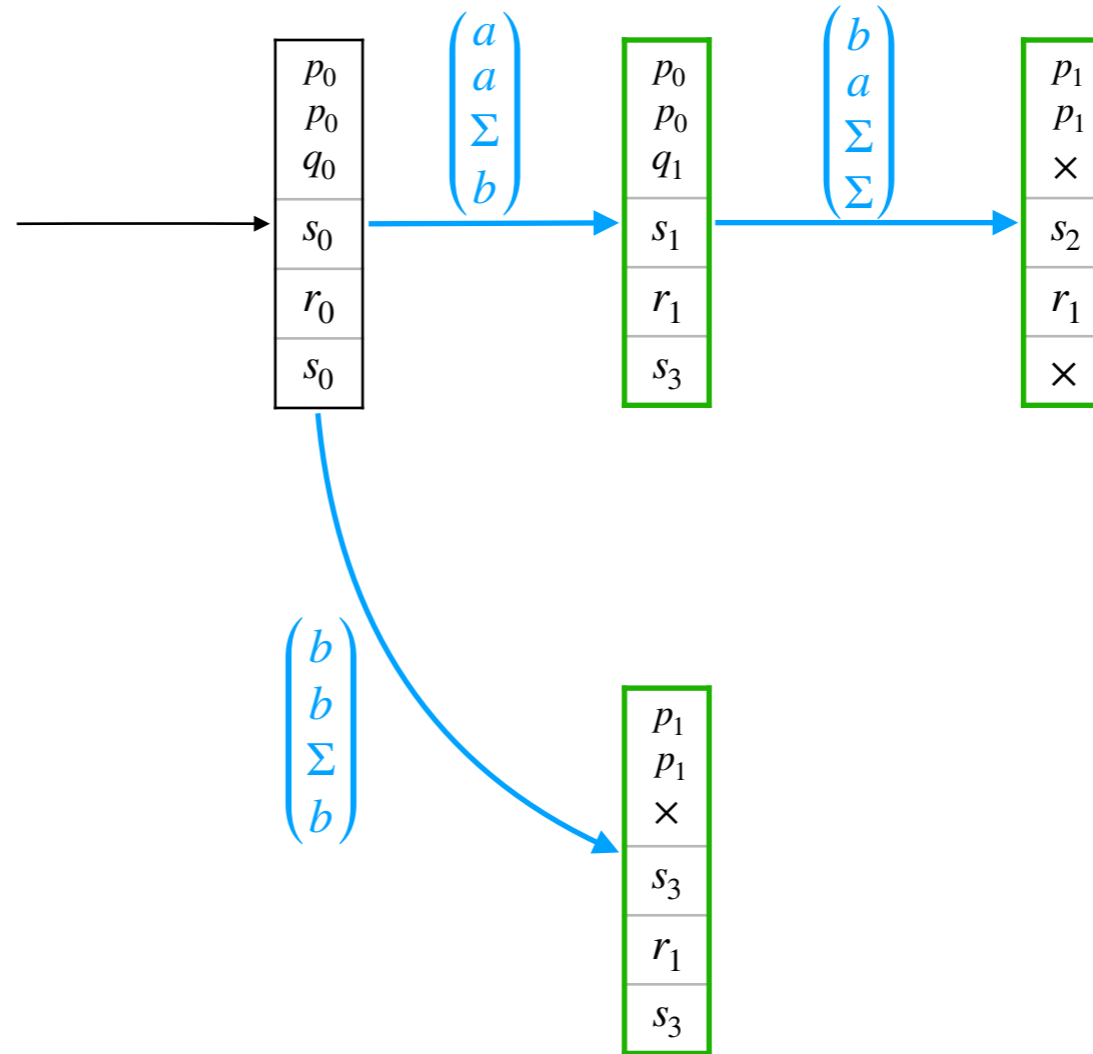
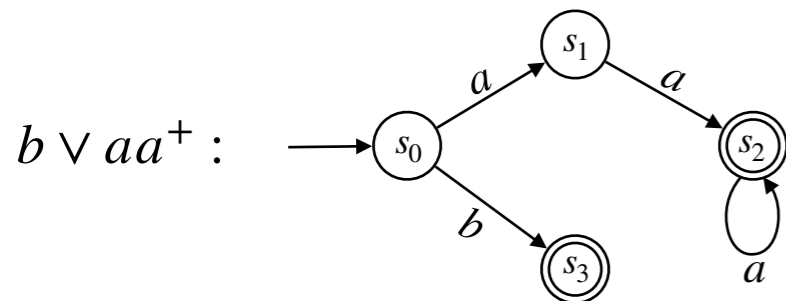
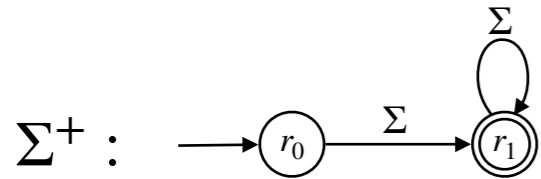
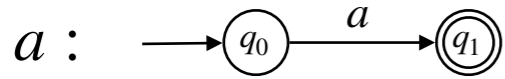
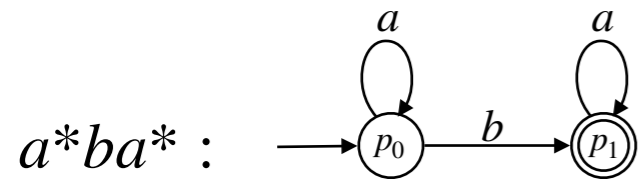
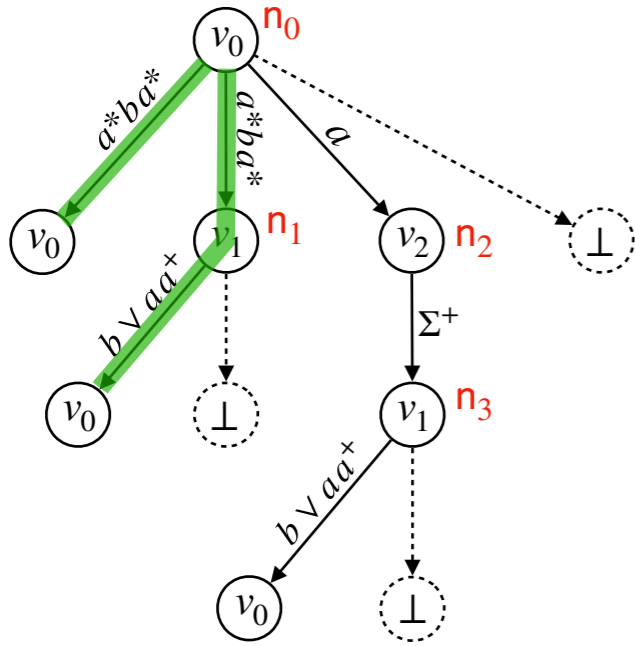
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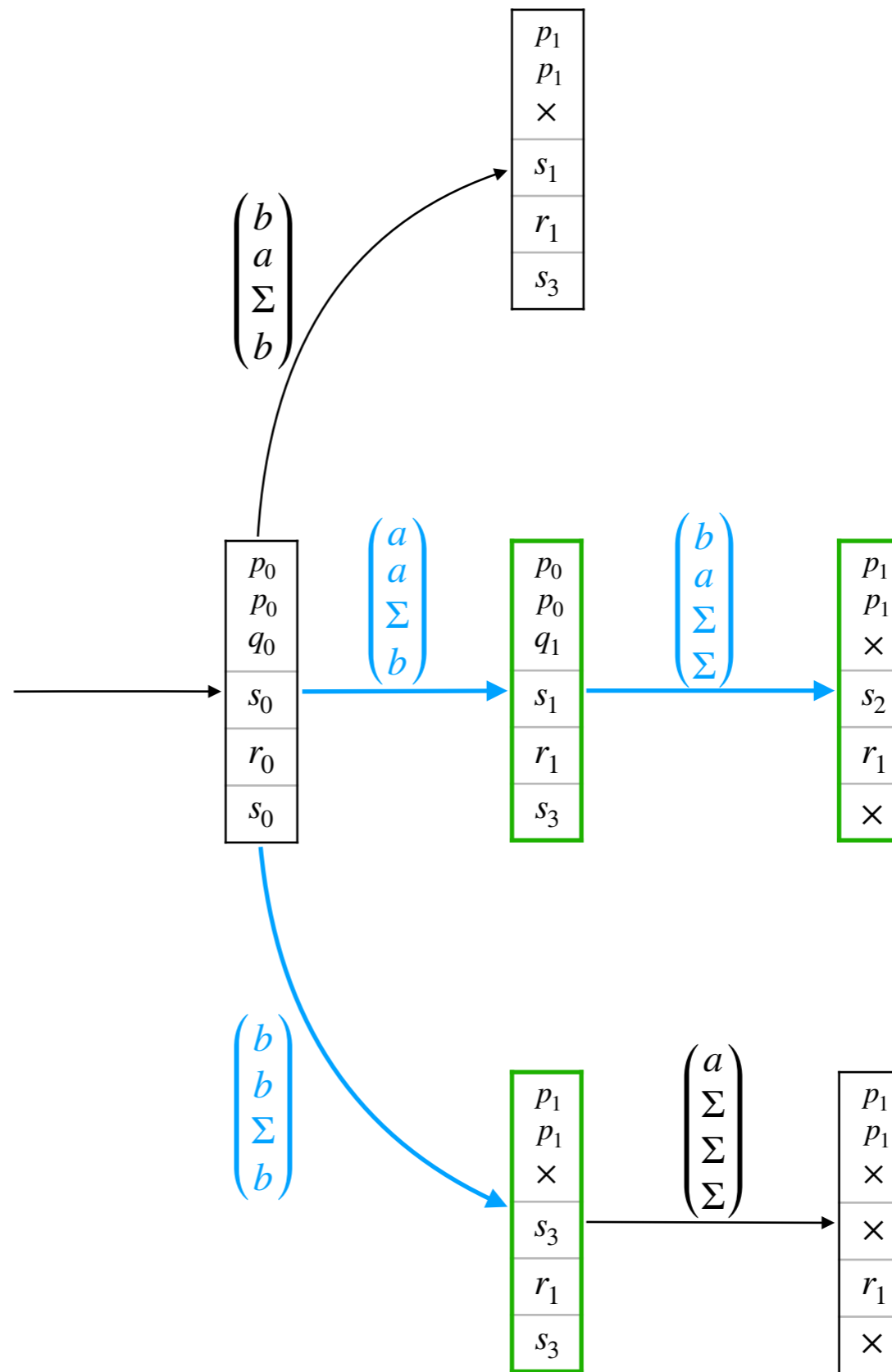
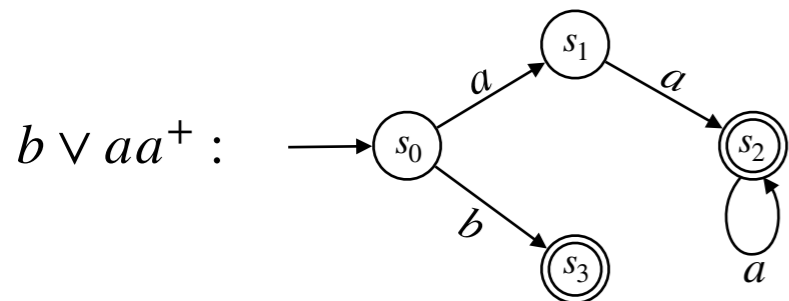
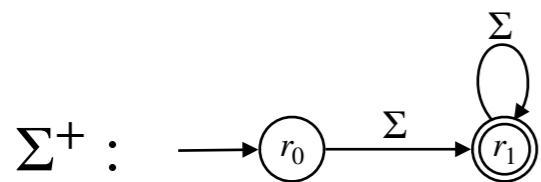
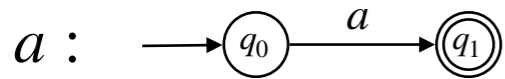
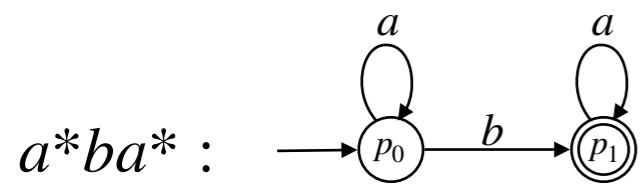
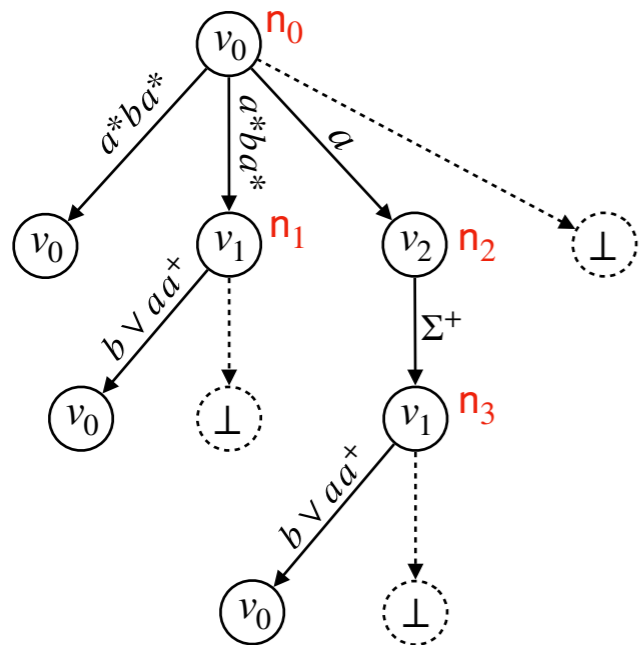
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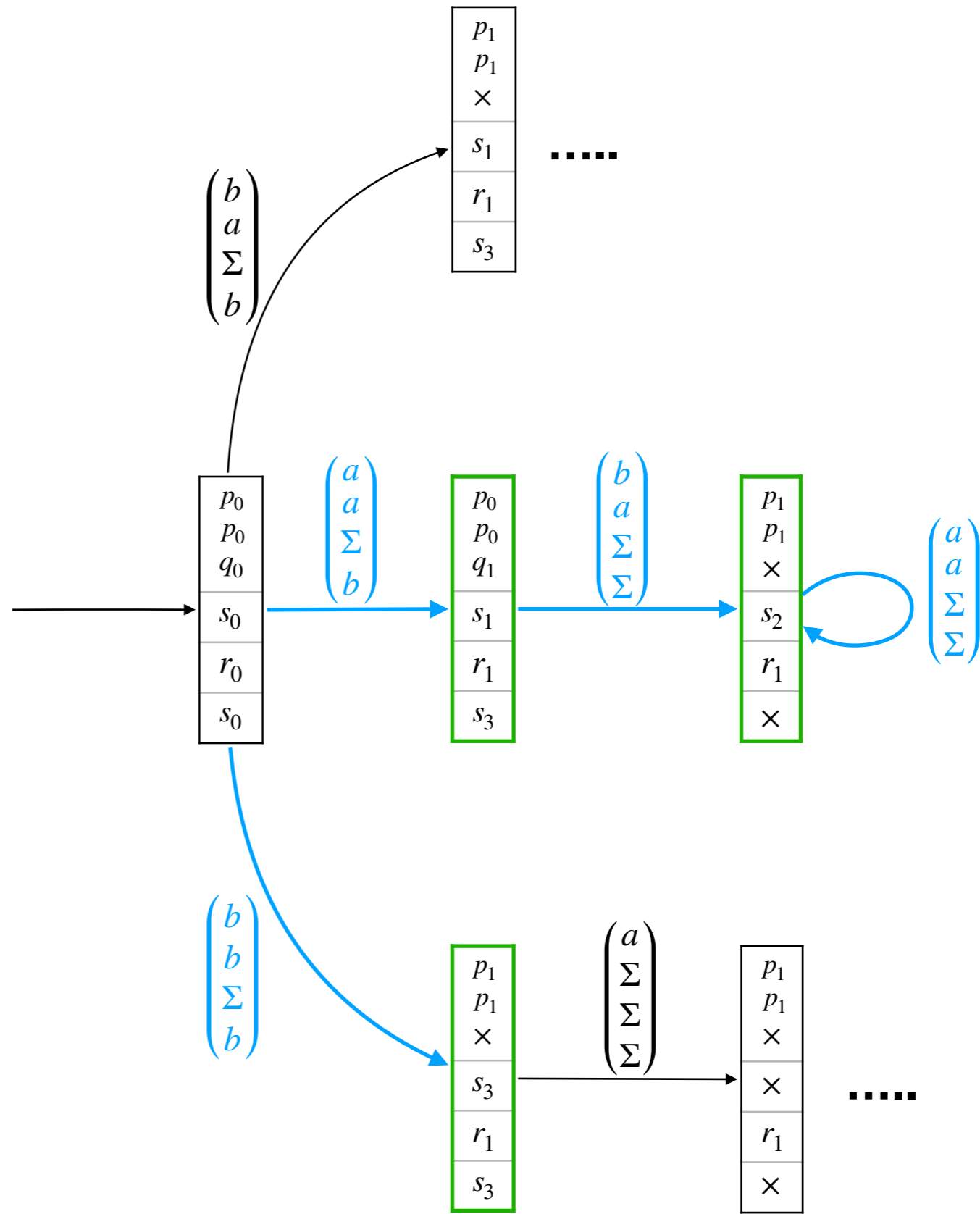
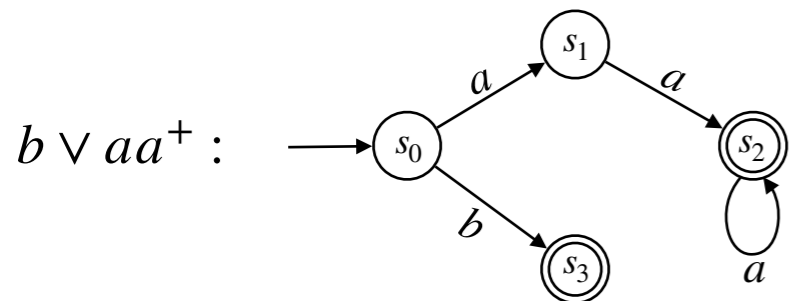
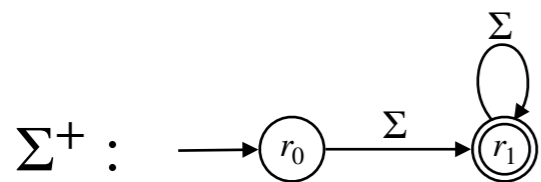
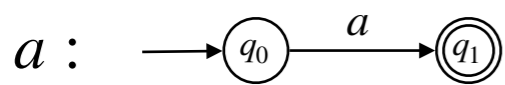
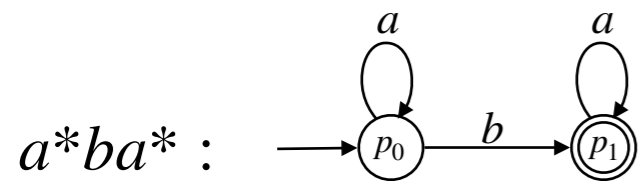
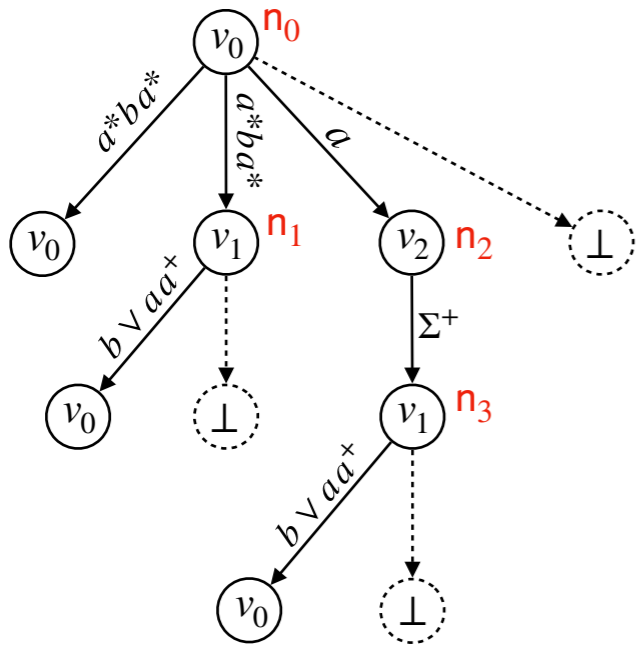
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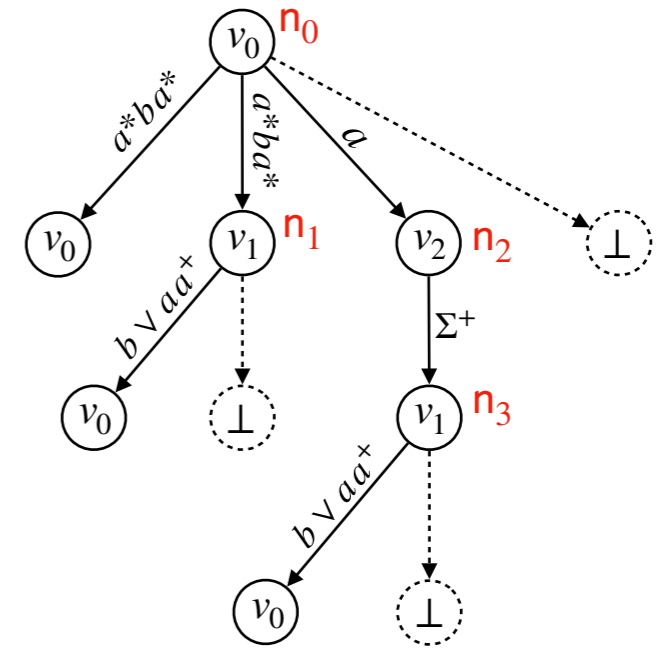


Synthesizing a winning strategy

► If $\mathcal{L}(\mathcal{B}) \neq \emptyset$, an **accepting** word of \mathcal{B} is $u \cdot v^\omega$.

► Define: $\lambda(n_i) = u_i \cdot v_i^\omega$

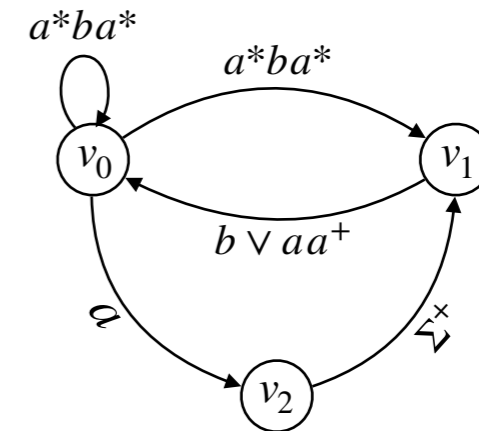
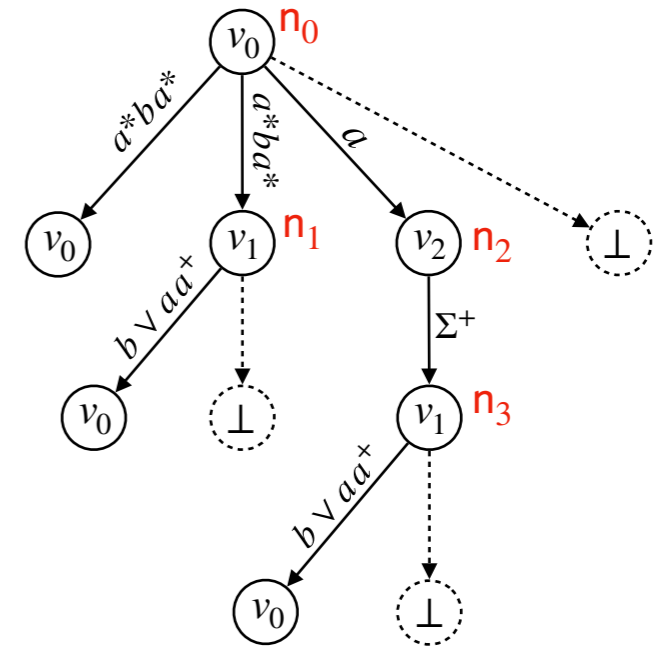
📌 λ is a winning strategy in \mathcal{T} .



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Transfer λ to a **winning** strategy $\tilde{\sigma}$ in \mathcal{G} :

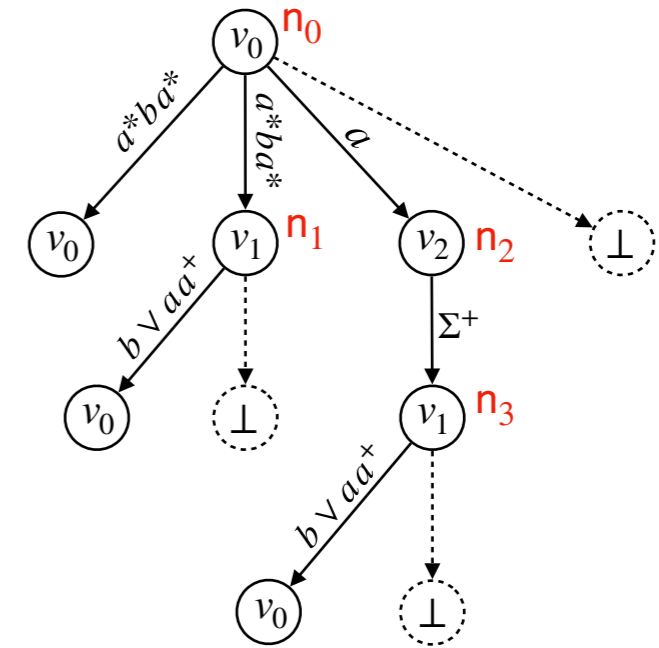


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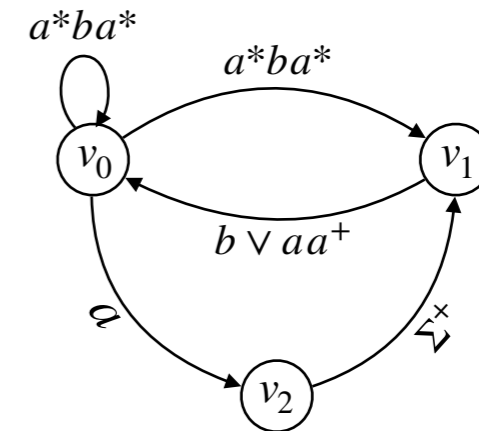


Transfer λ to a **winning** strategy $\tilde{\sigma}$ in \mathcal{G} :

► A history V^+ in \mathcal{A} **uniquely** maps to an internal node in \mathcal{T} by $h \mapsto \text{zip}(h)$.

► Define $\tilde{\sigma} : \tilde{\sigma}(h) = \lambda(\text{zip}(h))$

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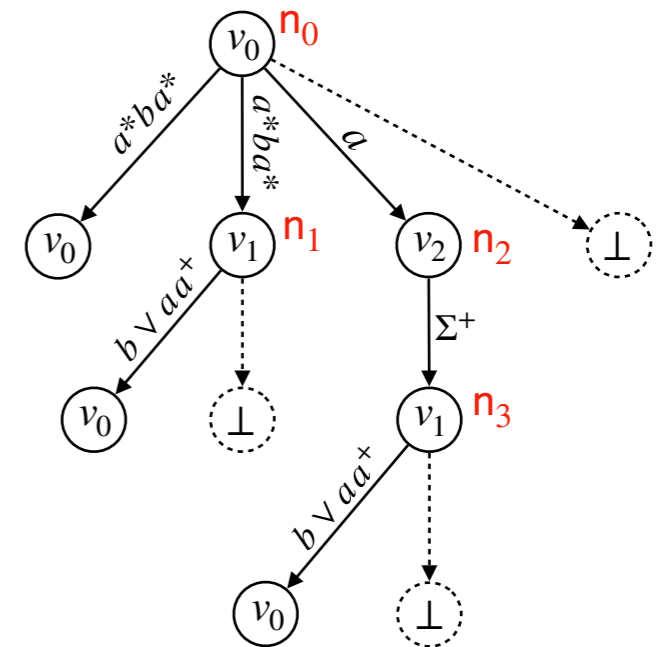


Synthesizing a winning strategy

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• λ is a winning strategy in \mathcal{T} .

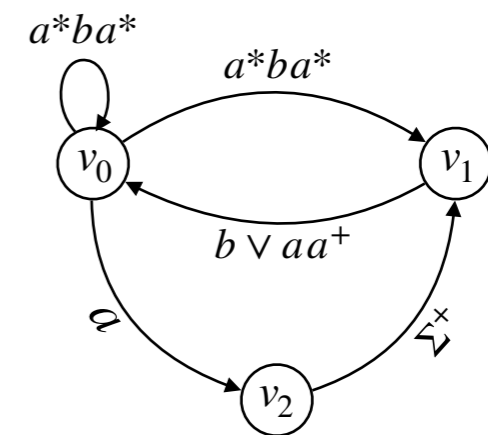


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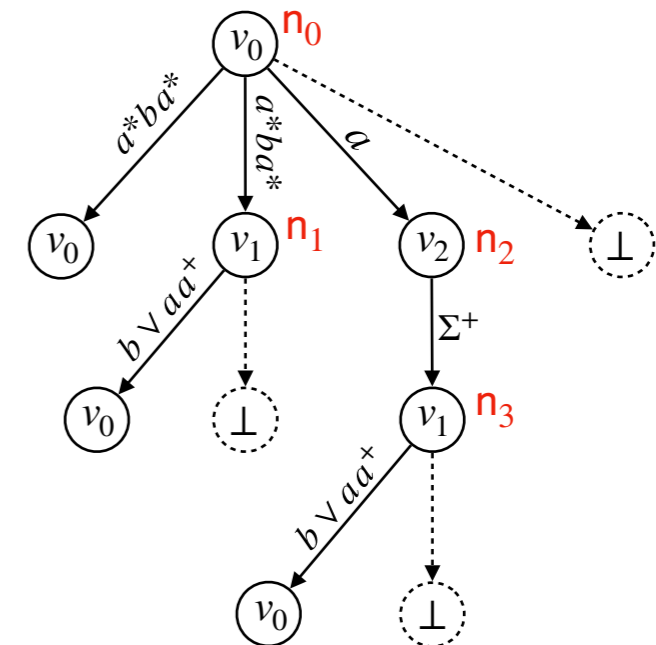
• $\tilde{\sigma}$ is a winning strategy in \mathcal{G} .



Synthesizing a winning coalition strategy is in **EXPSpace**.

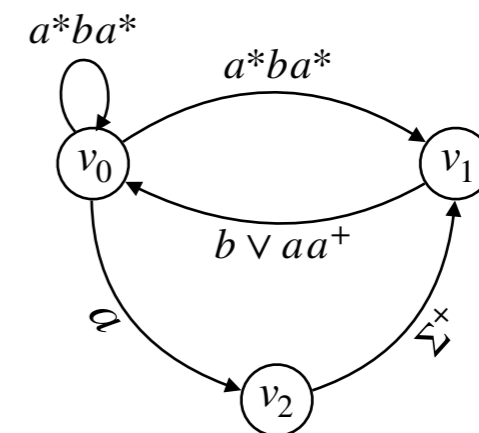
Synthesizing a winning strategy

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Transfer λ to a **winning** strategy $\tilde{\sigma}$ in \mathcal{G} :

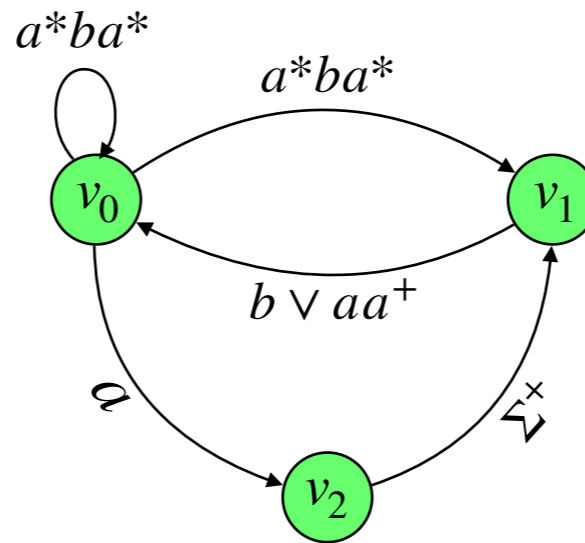
- ▶ A history V^+ in \mathcal{A} **uniquely** maps to an internal node in \mathcal{T} by $h \mapsto \text{zip}(h)$.
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Synthesizing a winning coalition strategy is in **EXPSpace**.

$\tilde{\sigma}$ uses memory of size $2^{O(|V|)}$, which is **unavoidable**.

Conclusion



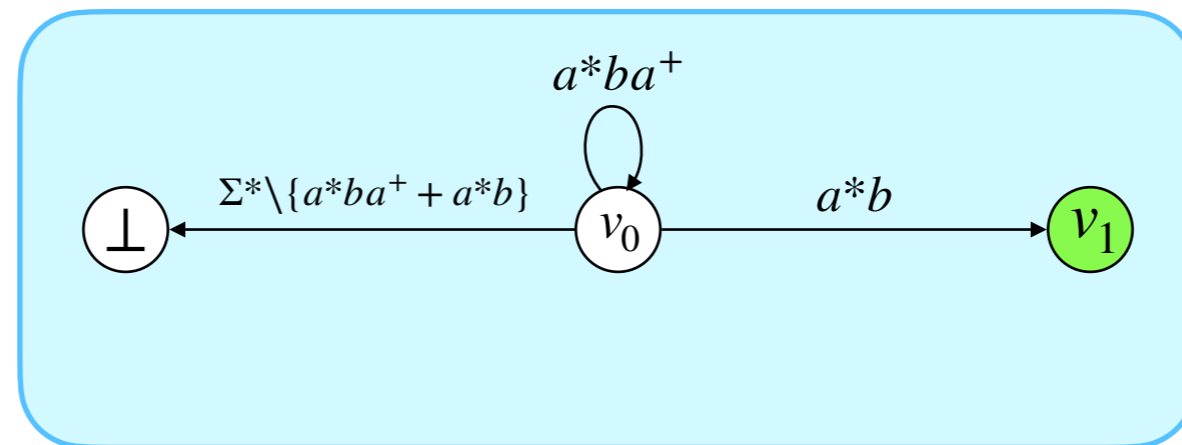
- ▶ Parameterized concurrent arena.
- ▶ Coalition problem with safety objective.
- ▶ Safe coalition problem is decidable in exponential space.
 - Reduce to coalition problem on finite tree unfolding.
 - Construct doubly-exponential size safety automaton.
 - Check non-emptiness.
- ▶ Safe coalition problem is PSPACE-hard.
- ▶ Synthesizing a winning strategy (if exists) needs exponential space.
- ▶ A winning strategy (if exists) needs exponential size memory.

Future work

▶ Tight complexity bound.

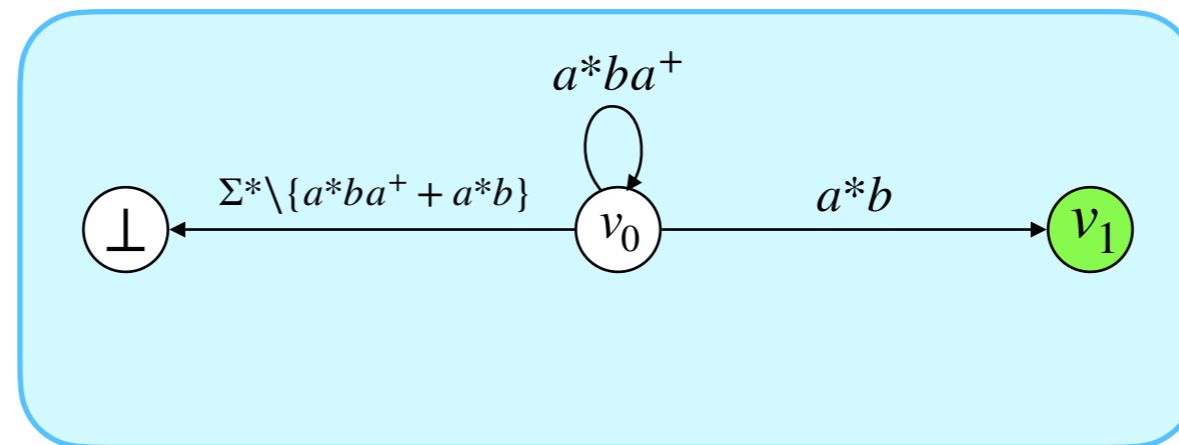
Future work

- ▶ Tight complexity bound.
- ▶ Reachability condition:



Future work

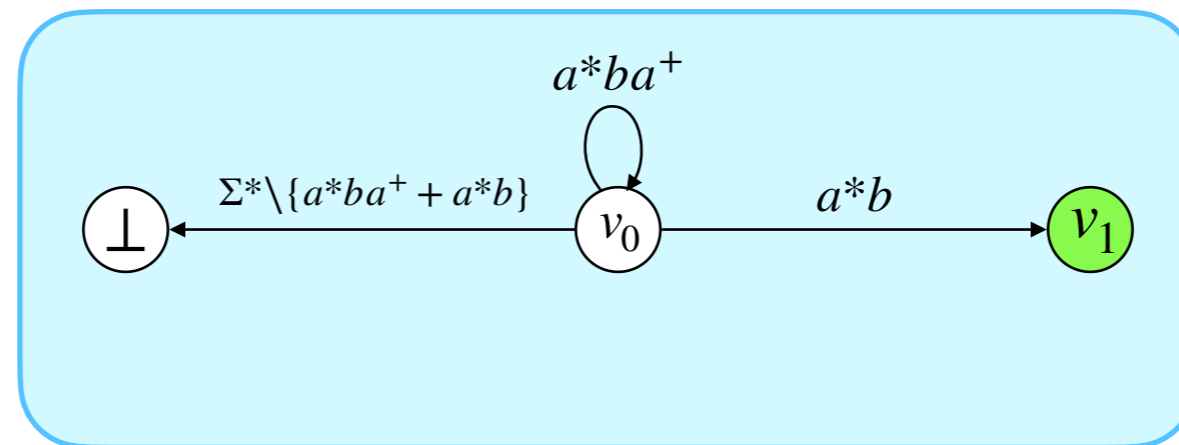
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- Players collectively want to **reach** v_1 .
- Number of players **unknown**.
- Collective winning strategy:
 - Player i plays b at i -th round, a otherwise.

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Thank You