Playing with Repeating Values in Datawords

Anirban Majumdar

M.Sc. Thesis

Supervised by: M Praveen

Chennai Mathematical Institute

June 14, 2018

Outline

Outline

Specifications for a coffee machine

- Whenever coffee button is pressed, coffee is produced in the next step.

$$\mathsf{G}(\text{coffee button} \Rightarrow \mathsf{X}(\text{coffee produced}))$$

Specifications for a coffee machine

- Whenever coffee button is pressed, coffee is produced in the next step.

$$\mathsf{G}(\text{coffee button} \Rightarrow \mathsf{X}(\text{coffee produced}))$$

- Whenever stop button is pressed, coffee is not produced in the next step.

$$\mathsf{G}(\text{stop button} \Rightarrow \mathsf{X}(\neg\ \text{coffee produced}))$$

Specifications for a coffee machine

- Whenever coffee button is pressed, coffee is produced in the next step.

$$\mathsf{G}(\text{coffee button} \Rightarrow \mathsf{X}(\text{coffee produced}))$$

- Whenever stop button is pressed, coffee is not produced in the next step.

$$\mathsf{G}(\text{stop button} \Rightarrow \mathsf{X}(\neg\ \text{coffee produced}))$$

- Specifications satisfiable:

coffee button	$\bot$	$\bot$	$\bot$	$\cdots$
stop button	$\bot$	$\bot$	$\bot$	$\cdots$
coffee produced	$\bot$	$\bot$	$\bot$	$\cdots$

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button
stop button

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button
stop button
coffee produced

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button $*$
stop button $*$
coffee produced

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button $*$
stop button $*$
coffee produced $*$

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$
stop button	$*$	$*$
coffee produced	$*$	

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$
stop button	$*$	$*$
coffee produced	$*$	$*$

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$	$*$
stop button	$*$	$*$	$*$
coffee produced	$*$	$*$	

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$	$*$
stop button	$*$	$*$	$*$
coffee produced	$*$	$*$	$*$

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$	$*$	$\cdots$
stop button	$*$	$*$	$*$	$\cdots$
coffee produced	$*$	$*$	$*$	

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$	$*$	$\cdots$
stop button	$*$	$*$	$*$	$\cdots$
coffee produced	$*$	$*$	$*$	$\cdots$

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$	$*$	$\cdots$
stop button	$*$	$*$	$*$	$\cdots$
coffee produced	$*$	$*$	$*$	$\cdots$

The realizability problem:

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$	$*$	$\cdots$
stop button	$*$	$*$	$*$	$\cdots$
coffee produced	$*$	$*$	$*$	$\cdots$

The realizability problem:
Input: A formula, a partition of the variables.

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$	$*$	$\cdots$
stop button	$*$	$*$	$*$	$\cdots$
coffee produced	$*$	$*$	$*$	$\cdots$

The realizability problem:
Input: A formula, a partition of the variables.
Question: Does the system have a winning strategy?

Realizability of specifications

Coffee button and stop button are not under the control of the system.

Two-player game: environment and system.

coffee button	$*$	$*$	$*$	$\cdots$
stop button	$*$	$*$	$*$	$\cdots$
coffee produced	$*$	$*$	$*$	$\cdots$

No winning strategy for system in the example.

The realizability problem:
Input: A formula, a partition of the variables.
Question: Does the system have a winning strategy?

Outline

Introduction of LRV

- Language over finite alphabet:
  - NFA, Buchi Automata.
  - LTL.

Introduction of LRV

- Language over finite alphabet:
  - NFA, Buchi Automata.
  - LTL.
- Language over infinite alphabet:

Introduction of LRV

- Language over finite alphabet:
  - NFA, Buchi Automata.
  - LTL.
- Language over infinite alphabet:
  - Register Automata, Data Automata.

Introduction of LRV

- Language over finite alphabet:
  - NFA, Buchi Automata.
  - LTL.
- Language over infinite alphabet:
  - Register Automata, Data Automata.
  - LRV.

Model, syntax, semantics

LRV: Extension of LTL

Model, syntax, semantics

LRV: Extension of LTL

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

Model, syntax, semantics

LRV: Extension of LTL

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

$$\begin{array}{lccccccc} x: & d_1 & * & * & \cdots & * & * & \cdots \\ y: & d_2 & * & * & \cdots & * & * & \cdots \\ q: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Model, syntax, semantics

LRV: Extension of LTL

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

$$\begin{array}{lccccccc} x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ q: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Model, syntax, semantics

LRV: Extension of LTL

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

$$\models$$

$$\begin{array}{lccccccc} x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ q: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Model, syntax, semantics

LRV: Extension of LTL

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

$$\begin{array}{lccccccc} & & \models & & & & \models \phi & \\ x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ q: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Model, syntax, semantics

LRV: Extension of LTL

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

$$\begin{array}{lccccccc} & & \models & & & \models \phi & & \\ x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ q: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Model, syntax, semantics

LRV: Extension of LTL

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

$$\begin{array}{lccccccc} & & \models \phi & & & & \models & \\ x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ q: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Model, syntax, semantics

LRV: Extension of LTL

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

$$\begin{array}{lccccccc} & \models \phi & & & & \models & & \\ x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ q: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Realizability of LRV formulas

- Realizability of propositional LTL: parity games on finite graphs.

Realizability of LRV formulas

- Realizability of propositional LTL: parity games on finite graphs.
- Satisfiability of LRV: reachability in VASS [Demri, D'Souza, Gascon 2007].

Realizability of LRV formulas

- Realizability of propositional LTL: parity games on finite graphs.
- Satisfiability of LRV: reachability in VASS [Demri, D'Souza, Gascon 2007].
- Realizability of LRV: parity games on VASS.

Parity games on VASS

- States partitioned into system and environment.

Parity games on VASS

- States partitioned into system and environment.
- Finite set of counters.

Parity games on VASS

- States partitioned into system and environment.
- Finite set of counters.
- Transitions increments or decrements a counter or does not effect any counter.

Parity games on VASS

- States partitioned into system and environment.
- Finite set of counters.
- Transitions increments or decrements a counter or does not effect any counter.
- A player can choose a decrementing transition only if the counter has non-zero value.

Parity games on VASS

- States partitioned into system and environment.
- Finite set of counters.
- Transitions increments or decrements a counter or does not effect any counter.
- A player can choose a decrementing transition only if the counter has non-zero value.
- System wins an infinite play if it satisfies the parity condition.

Outline

Asymmetry in games on VASS

- [Raskin, Samuelides, Van Begin 2005] One of the palyers has transitions that are downward closed. Coverability games decidable.
- [Abdulla, Bouajjani, D'orso 2008] One of the players has lossy transitions. Safety games are decidable.
- [Brázdil, Jančar, Kučera 2010] Transitions can add arbitrarily large numbers. Decidable to check if one of the players can make some counter zero.
- [Bérard, Haddad, Sassolas, Sznajder 2012] One palyer can only increment; the other player cannot test for zero.
- [Chatterjee, Randour, Raskin 2013] Energy games: if a player makes a counter to go below zero, the other player wins immediately. One of the players has to additionally satisfy a parity condition.

Asymmetry in games on VASS

- [Raskin, Samuelides, Van Begin 2005] One of the palyers has transitions that are downward closed. Coverability games decidable.
- [Abdulla, Bouajjani, D'orso 2008] One of the players has lossy transitions. Safety games are decidable.
- [Brázdil, Jančar, Kučera 2010] Transitions can add arbitrarily large numbers. Decidable to check if one of the players can make some counter zero.
- [Bérard, Haddad, Sassolas, Sznajder 2012] One palyer can only increment; the other player cannot test for zero.
- [Chatterjee, Randour, Raskin 2013] Energy games: if a player makes a counter to go below zero, the other player wins immediately. One of the players has to additionally satisfy a parity condition.
- [Abdulla, Mayr, Sangnier, Sproston 2013] Single-sided VASS games: Environment cannot change counter value.

Single-sided LRV games

- Environment player has only Boolean variables.

Single-sided LRV games and More Resrtictions

- Environment player has only Boolean variables.
- No nested formulas: only $x \approx \langle \top ? \rangle^{-1} y$.

Single-sided LRV games and More Resrtictions

- Environment player has only Boolean variables.
- No nested formulas: only $x \approx \langle \top ? \rangle^{-1} y$.
- No future obligations: $x \approx \langle \top ? \rangle y$ not allowed.

Single-sided LRV games and More Resrtictions

- Environment player has only Boolean variables.
- No nested formulas: only $x \approx \langle \top ? \rangle^{-1} y$.
- No future obligations: $x \approx \langle \top ? \rangle y$ not allowed.
- Realizability can be reduced to single-sided VASS games: Decidable.

Single-sided LRV games and More Resrtictions

- Environment player has only Boolean variables.
- No nested formulas: only $x \approx \langle \top ? \rangle^{-1} y$.
- No future obligations: $x \approx \langle \top ? \rangle y$ not allowed.
- Realizability can be reduced to single-sided VASS games: Decidable. [Diego Figueira, M Praveen 2018].

Single-sided LRV games and More Resrtictions

- Environment player has only Boolean variables.
- Nested formulas: $x \approx \langle \phi ? \rangle^{-1} y$; where ϕ depends only on past.
- No future obligations: $x \approx \langle \top ? \rangle y$ not allowed.
- Realizability can be reduced to single-sided VASS games: Decidable. [This Thesis]

Single-sided LRV games — symbolic models

Concrete model

$$\models y \approx \Diamond^{-1} x$$

$$\downarrow$$

$$\begin{array}{lccccccc} x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ w: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Single-sided LRV games — symbolic models

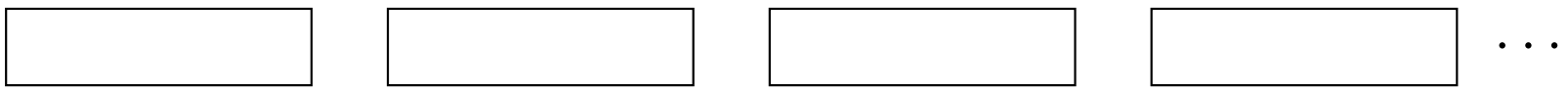
Concrete model

$$\models y \approx \Diamond^{-1} x$$

$$\downarrow$$

$$\begin{array}{lccccccc} x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ w: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Symbolic model



Single-sided LRV games — symbolic models

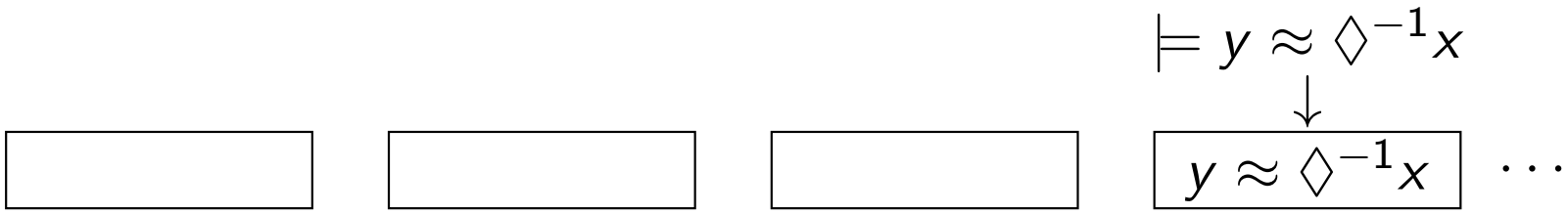
Concrete model

$$\models y \approx \Diamond^{-1} x$$

$$\downarrow$$

$$\begin{array}{lccccccc} x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ w: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Symbolic model



Single-sided LRV games — symbolic models

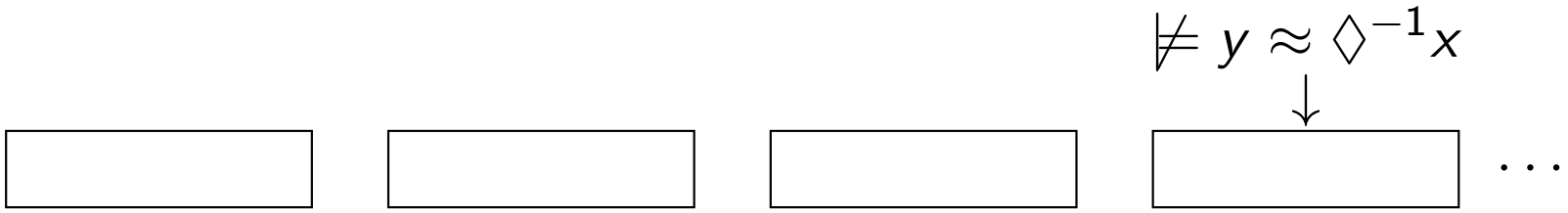
Concrete model

$$\models y \approx \Diamond^{-1} x$$

$$\downarrow$$

$$\begin{array}{lccccccc} x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ w: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Symbolic model



Single-sided LRV games — symbolic models

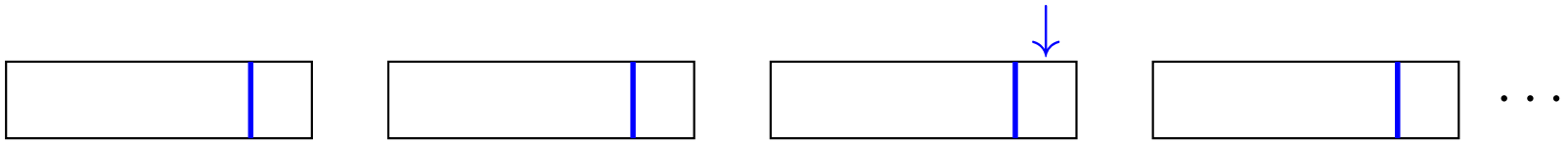
Concrete model

$$\models y \approx \langle \phi ? \rangle^{-1} x$$

$$\begin{array}{lccccccc} x: & d_1 & d & * & \cdots & * & * & \cdots \\ y: & d_2 & * & d & \cdots & d' & d & \cdots \\ w: & \top & \bot & \top & \cdots & * & * & \cdots \end{array}$$

Symbolic model

Extra information about past positions having same data value



Symbolic models

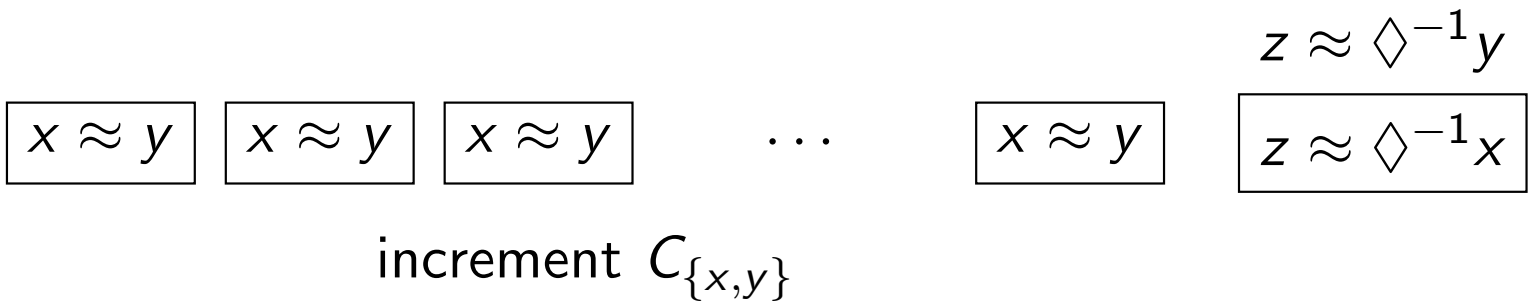
$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

Symbolic models

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$

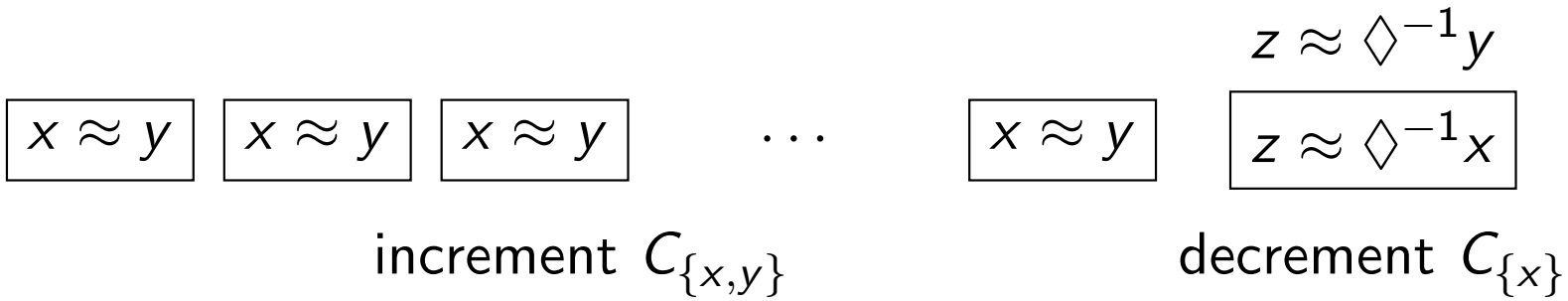
Symbolic models

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$



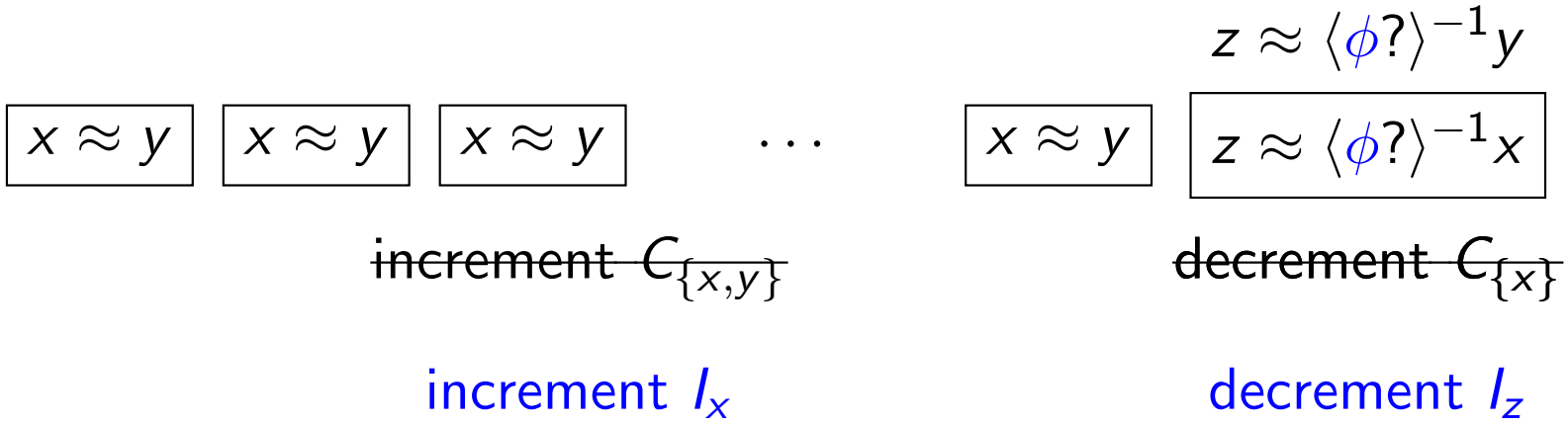
Symbolic models

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$



Symbolic models

$$\phi ::= q \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi \mid \phi \mathsf{S} \phi \mid$$
$$x \approx \mathsf{X}^1 y \mid x \approx \langle \phi ? \rangle y \mid x \not\approx \langle \phi ? \rangle y \mid y \approx \langle \phi ? \rangle^{-1} x \mid y \not\approx \langle \phi ? \rangle^{-1} x$$



Outline

Restrictions to get decidability

- No nested formulas: only $x \approx \langle \top ? \rangle^{-1} y$.
- Environment player has only Boolean variables.
- No future obligations: $x \approx \langle \top ? \rangle y$ not allowed.
- Realizability: Decidable.

Restrictions to get decidability

- Nested formulas: $x \approx \langle \phi ? \rangle^{-1} y$; where ϕ depends only on past.
- Environment player has only Boolean variables.
- No future obligations: $x \approx \langle \top ? \rangle y$ not allowed.
- Realizability: Decidable. [This thesis]

Not decidable anymore

- Nested formulas: $x \approx \langle \phi ? \rangle^{-1} y$.
- Environment player has only Boolean variables.
- No future obligations: $x \approx \langle \top ? \rangle y$ not allowed.
- Realizability: Undecidable. [This thesis]

Lossy Counter Machines

- LCM: Counter Machine + Lossiness.

Lossy Counter Machines

- LCM: Counter Machine + Lossiness.
- Counter Machine: Transitions can either increment, decrement or test for value zero of a counter.

Lossy Counter Machines

- LCM: Counter Machine + Lossiness.
- Counter Machine: Transitions can either increment, decrement or test for value zero of a counter.
- Lossiness: Sum of counter values may decrease in every transition.

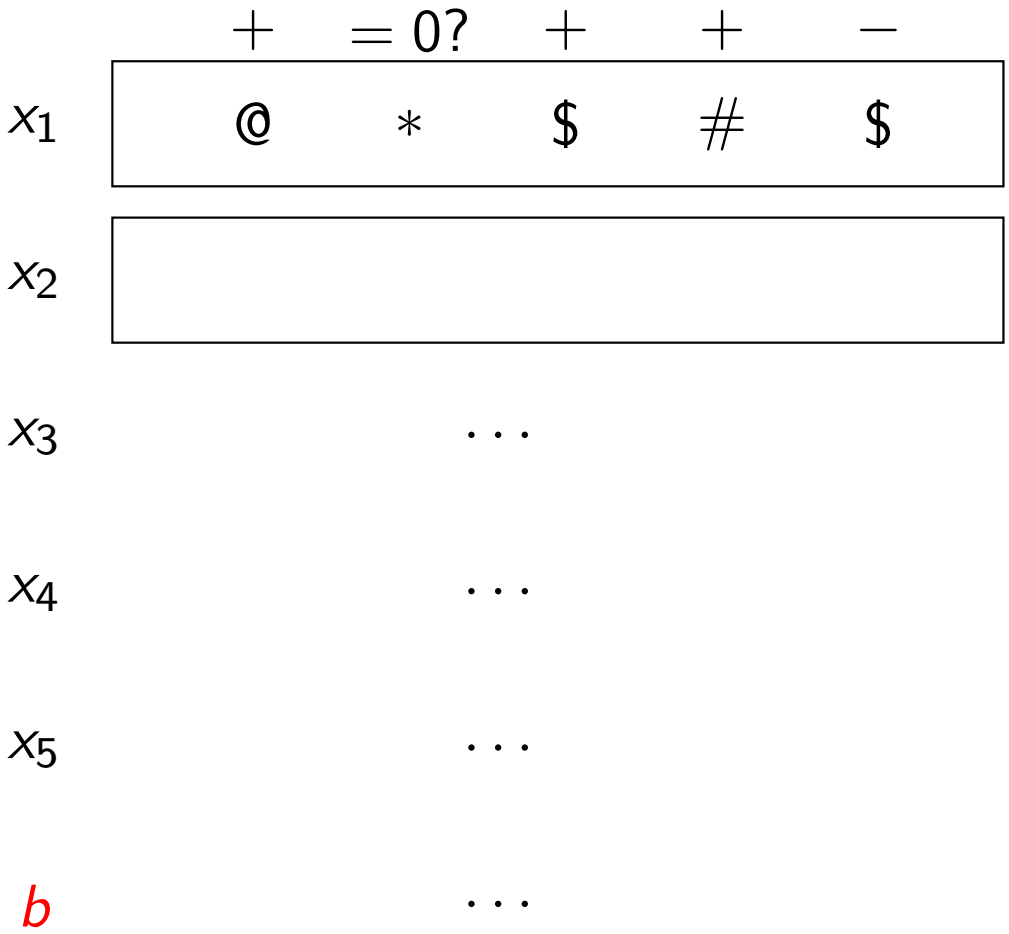
Lossy Counter Machines

- LCM: Counter Machine + Lossiness.
- Counter Machine: Transitions can either increment, decrement or test for value zero of a counter.
- Lossiness: Sum of counter values may decrease in every transition.
- Reset Lossiness: At any zero test transition, the corresponding counter value can immediately become zero.

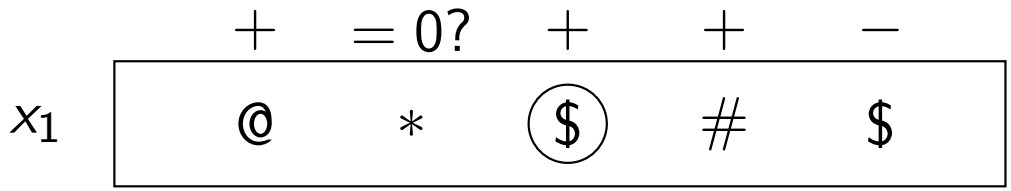
Lossy Counter Machines

- LCM: Counter Machine + Lossiness.
- Counter Machine: Transitions can either increment, decrement or test for value zero of a counter.
- Lossiness: Sum of counter values may decrease in every transition.
- Reset Lossiness: At any zero test transition, the corresponding counter value can immediately become zero.
- Checking the existense of a configuration from which there is an infinite run in a 5-counter LCM: Undecidable. [Richard Mayr, 1998]

Simulating counter machines

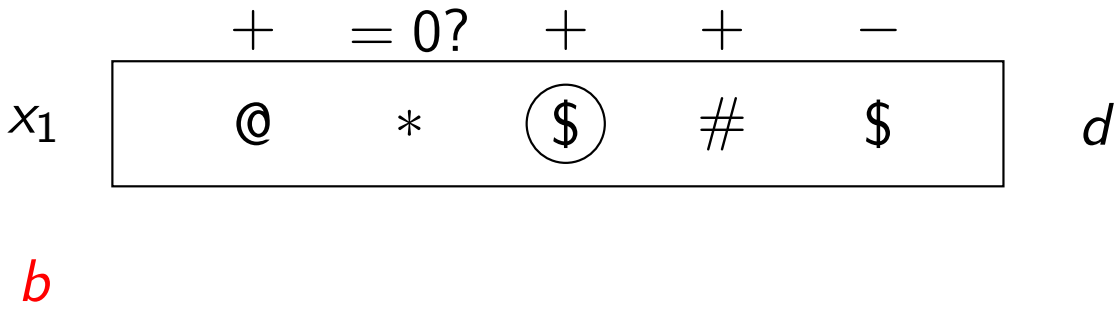


Simulating counter machines



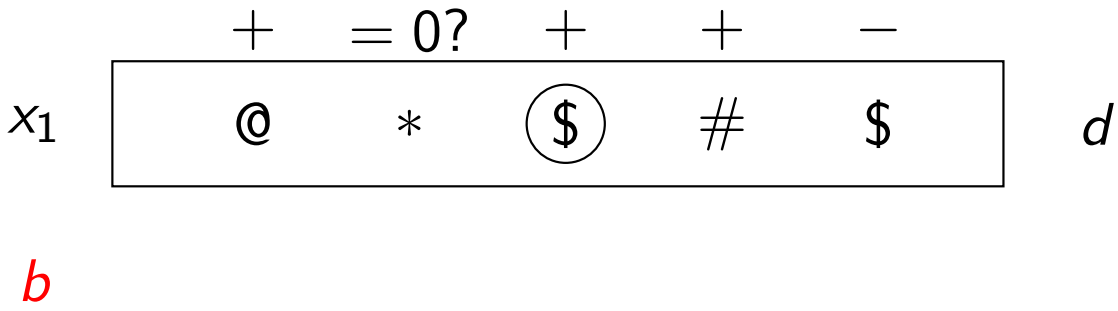
b

Simulating counter machines



Increment

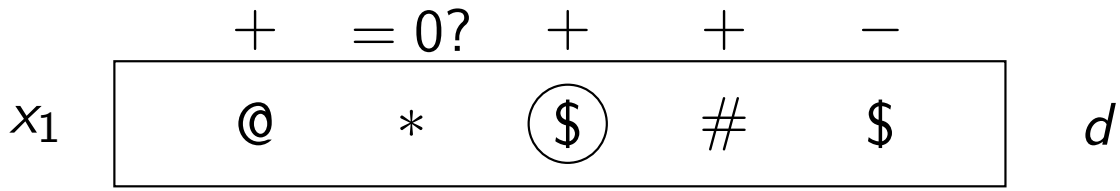
Simulating counter machines



Increment

- d should be a new data value.

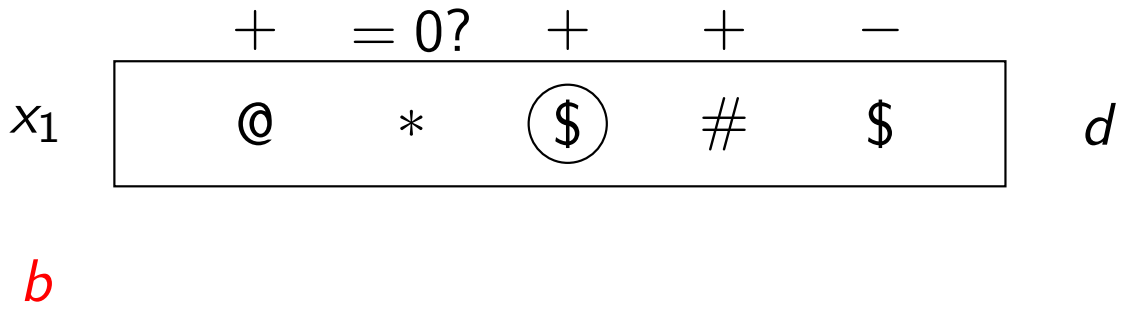
Simulating counter machines



b

Zero test

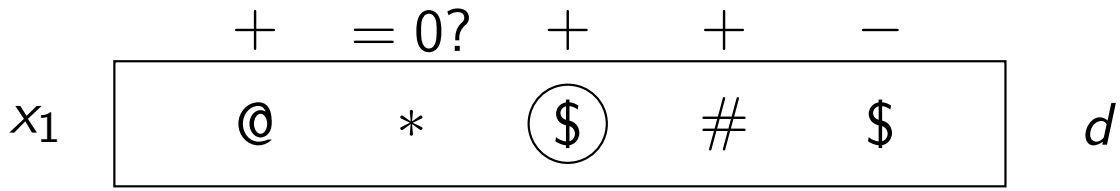
Simulating counter machines



Zero test

- Counter can immediately goes to zero; wlog, d is a new data value.

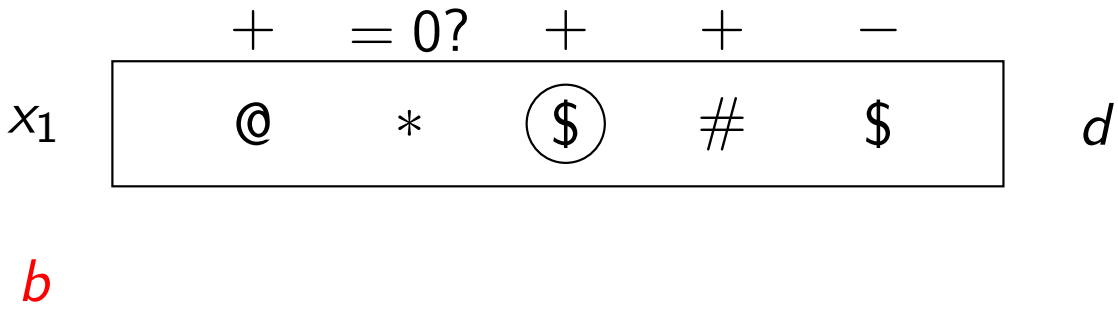
Simulating counter machines



b

Decrement

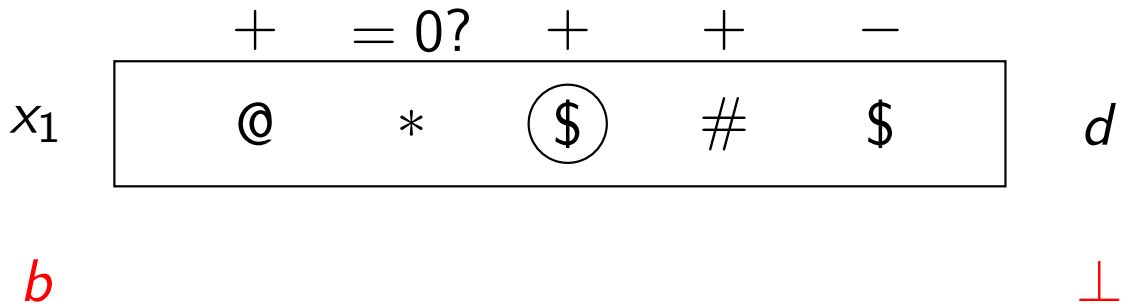
Simulating counter machines



Decrement

- d must repeat in the past in an incrementing position and no zero test in between.

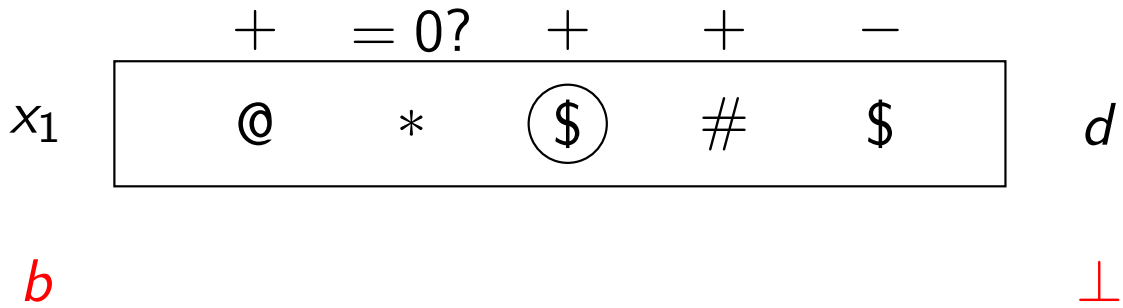
Simulating counter machines



Decrement

- d must repeat in the past in an incrementing position and no zero test in between.
- If not, second player sets b to false.

Simulating counter machines



Decrement

- d must repeat in the past in an incrementing position and no zero test in between.
- If not, second player sets b to false.
- System should justify he is not cheating: can be captured by a formula in this fragment of LRV.

Outline

Conclusion

- LRV Realizability.

Conclusion

- LRV Realizability.
- Undecidable in the general case.

Conclusion

- LRV Realizability.
- Undecidable in the general case.
- Restrictions to get decidability: single-sided game, no future obligations, no nested formula.

Conclusion

- LRV Realizability.
- Undecidable in the general case.
- Restrictions to get decidability: single-sided game, no future obligations, no nested formula.
- Single-sided, no future obligations, nested formulas depends on past: Decidable.

Conclusion

- LRV Realizability.
- Undecidable in the general case.
- Restrictions to get decidability: single-sided game, no future obligations, no nested formula.
- Single-sided, no future obligations, nested formulas depends on past: Decidable.
- Single-sided, no future obligations, nesting is allowed: Undecidable.

Outline

Future work

- Tight bound on nesting.
- Complexity bounds.
- Synthesizing winning strategies.
- Other decidable restrictions of VASS games.

Future work

- Tight bound on nesting.
- Complexity bounds.
- Synthesizing winning strategies.
- Other decidable restrictions of VASS games.

Thank you